this time: confidence intervals

DS ch. 8 sec 8.5

STAT 131 5 June 20

Catch-up lecture

population

all U.S. families in 2009 with income that year ≤ \$8m

yearly income

$N = ?$ (big) \$200m

pop. mean $\mu = ?$

pop. SD $\sigma = ?$

pop. PDF of X_i

sample

the observed families

yearly income, 2009, in \$k

actual IID

X_1 x_1 = \$72k

X_i 29k

330k

X_n

$n = 842$

sample mean $\bar{x}$ = \$82.9k

sample SD s = \$100.1k

sample skewness = 5.1

descriptive summaries

sample excess kurtosis = 33.8

0

sample hist

hyp. IID

$n = 842$

mean $\bar{x} = ?$ (ex. \$86.8k)

~~repeated~~ sampling at least 1 possible $\bar{x}$s

\$82.9k

\$86.8k

(big) $M \to \infty$

long run mean

(WLLN)

$E_{IID}(\bar{X}) = \mu$ ✓

estimated long run SD

$\widehat{SD}(\bar{X})$ = \$3.5k

$\widehat{SE}(\bar{X}) = \frac{\$s}{\sqrt{n}}$

PDF of $\bar{X}$

long run hist

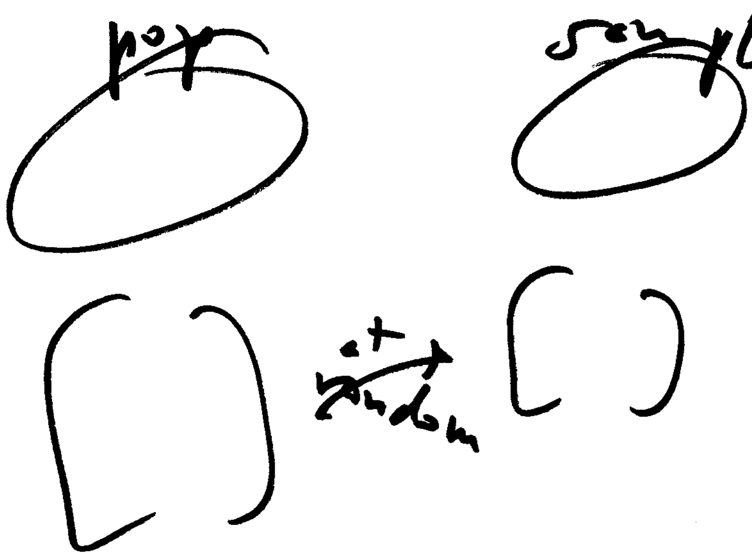
CLT

inferential summary

repeated-sampling sample of pop

unknown pop. quantity of main interest	μ = pop. mean U.S. annual family income in 2009 (\$M)
estimate of μ	$\bar{x}$ = \$82.9K
give or take for $\bar{x}$ as est. of μ	$\hat{SE}(\bar{X}) = \frac{s}{\sqrt{n}} \doteq \$3.5K$
99.9% CI for μ	(\$71.5K, \$94.2K)

CI: confidence interval



if pop known → ?	probability (easier) STAT 131
? ← if sample known	statistical (harder) inference STAT 132, 206

On the basis of this IID sample of size $n = 842$, we think

that μ is around $\underline{\bar{x} = \$82.9k}$,

give or take about $\underline{\widehat{SE}(\bar{X}) = \$3.5k}$,

and we're 99.9% confident (given that our assumptions are correct)

that μ is $\underline{\text{between } \$71.5k \text{ and } \$94.2k}$.

how measure how accurate $\bar{X}$ is as an estimate of μ?
(r.v.)

$$P_{\text{F}}\left(\underbrace{|\bar{X} - \mu|}_{\text{want small}} \le \text{"small } \epsilon\text{"} \right) = \text{big (close to 1)}$$

F = frequentist (Mr. Neyman)

$\boxed{\bar{X} \text{ estimates } \mu}$ so frequentists

use a different term for $SD(\bar{X})$:

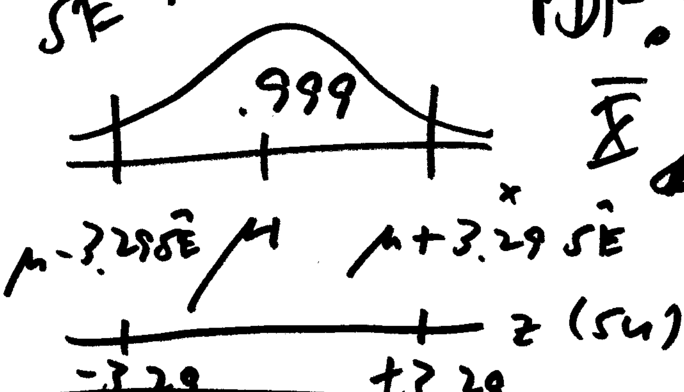
~~estimated~~ $\cancel{= \hat{SD}_{IID}(\bar{X})}$

$$\left(\begin{array}{l}\text{estimated}\\ \text{standard}\\ \text{error}\\ \text{of } \bar{X}\end{array}\right) = \hat{SE}_{IID}(\bar{X}) = \sqrt{\hat{V}_{IID}(\bar{X})}$$

$$= \frac{\cancel{\sigma}\, s}{\sqrt{n}} = \frac{\$100.1K}{\sqrt{842}} = \$3.45K$$

$\hat{SE}$ \$3.5K

PDF of $\bar{X}$



$100(1-\alpha)\%$ CI for μ

(\$71.5K $\bar{X}$ \$94.2K)

$\alpha = .001$

$$P_F\left(\mu - 3.29\hat{SE} < \bar{X} < \mu + 3.29\hat{SE}\right) = 0.999$$

Fisher: (1925)

~~95%~~ 99% ~~99.5~~ ~~99.7 (3)~~

99.9%

$$P_F\left(\bar{X} - 3.29\hat{SE} < \mu < \bar{X} + 3.29\hat{SE}\right) = 0.999$$

Mr. N's proposal: use $\bar{X} \pm 3.29\, \hat{SE}(\bar{X})$ as a $100(1-\alpha)\%$ confidence interval for μ ($\alpha = 0.001$)

$$\boxed{\bar{X} \pm \Phi^{-1}\left(1-\frac{\alpha}{2}\right)\frac{s}{\sqrt{n}}}$$

← $100(1-\alpha)\%$ CI, n large

99.9% CI for μ

($71.5k — $\bar{X}$ $82.9k — $94.2k)

person 1: I think μ that year was $70k

p_2 (I think μ was $100k)

no, data doesn't support that theory

p_3 (I think μ was $80k)

data does support that theory, (+)

but we think that μ is closer to \$83k than \$90k

\$83.5k

$p1$

$\mu = \$70k$
μ_0

~~99.9% CI for μ~~

~~(|)~~

\$71.5k \$82.9k \$94.2k

Since \$70k is not in our 99.9% CI for μ, the difference between $\bar{x} = \$82.9k$ and $\mu_0 = \$70k$ is statistically significant (statsig) (at the 99.9% confidence level)

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p_3 : $\mu = \$90k$
μ_0

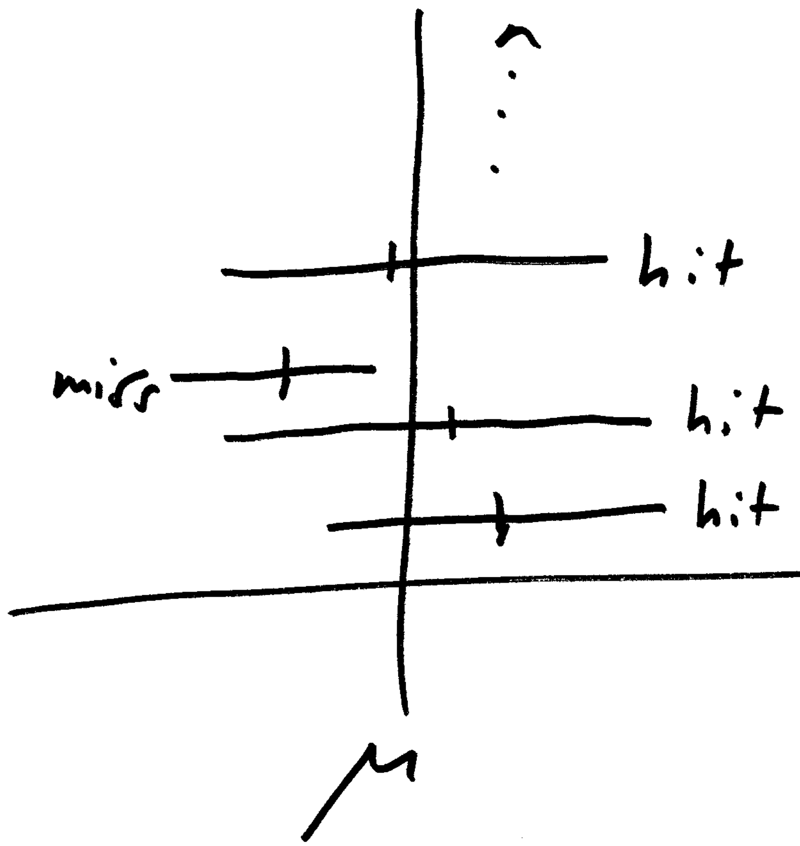
no statsig diff. between $\bar{x}$ and μ_0.

(diff is statsig) $\longleftrightarrow$ (hard to attribute diff. to unlucky random sampling)

$\leftrightarrow$ (diff. is probably real)

$$P_F(\$71.5k < \mu < \$94.2k) \neq 0.999$$

fixed unknown constant



Neyman: you will achieve about a 99.9% hit rate with my method