STAT 131
31 Mar 20

this time: basic probability rules

next time: foundations

course textbook: DeGroot & Schervish (2012), in private (DS) section of course ①

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webpage stat131-spring20-01.courses.soe.ucsc.edu

(you need to login to this webpage to access the textbook PDF)

read: DG ch. 1

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assessment: 10 quizzes (1/week) and 3 take-home tests (THT)

all quizzes & THTs will be open-book, open-notes with moderate time pressure; you will upload PDF versions of your solutions to Canvas in time to meet deadlines

This course is FAST and CUMULATIVE;
try hard not to fall behind.

Pascal & Fermat (1654): classical "approach to probability

Chevalier de Méré

Equally Likely Model (ELM)

If you can list all the ways a random experiment can come out in such a way that no outcome is favored over another, (elemental outcomes, EOs)

$$P_{PF}(\underline{A}) = \frac{\text{\# of EOs favorable to } A}{\text{total \# of EOs}}$$

Ex. single six-sided die:

dice-rolling **fair** : all 6 EOs $\{1, 2, \ldots, 6\}$

are equally likely/plausible

if fair, $P_{PF}(\#\text{ is at least } 5) = \dfrac{2}{6} = \dfrac{1}{3}$

$\{5, 6\} \to 2$

$P_{PF}\left(\#\text{ is } \geq 5 \;\middle|\; \text{die-rolling is fair}\right) =$

"given"

$\{1, \ldots, 6\}$

$P_{PF}(\#\text{ is } \geq 5) = \text{undefined}$

this is one of three ways to define probability

1. P-F (1654)
2. Bayesian (1750)
3. Frequentist (1825)

(2) $P_B(C \mid A)$ ← assumptions = a measure of the strength or weight of evidence that C is true, given that assumptions in A are true

consequences

Thomas Bayes (1763)

C = true/false statement

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(3) frequentist (relative frequency)

P_F (H on a single toss | fair) = ?

H T H H T H T T T

$1 \quad \frac{1}{2} \quad \frac{2}{3} \quad \frac{3}{4} \quad \frac{3}{5} \quad \frac{4}{6} \quad \ldots \rightarrow$ unique limit $= \frac{1}{2}$

$0 \le P_B(C \mid \underline{A}) \le 1$

T/F statements

F

	H	h
H	HH	Hh
h	hH	hh

M

normal

carrier

T-S

4

$P_{PF}(\text{normal} \mid F, M \text{ carriers}) = \frac{1}{4} = 25\% = 0.25$

ELM? Yes

$P_{PF}(\text{carrier} \mid F, M \text{ carriers}) = \frac{2}{4} = 50\% = 0.5$

$P_{PF}(T\text{-}S \mid F, M \text{ carriers}) = \frac{1}{4} = 25\% = 0.25$

# children	P(1 or more T-S / F, M carriers)
1	0.25
2	
⋮	
5	?

as # children ↑, P( ) ↑

6

population

$$\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \xrightarrow{\text{at random}} \begin{bmatrix} Y_1 \end{bmatrix}$$

sample

ELM? yes

$P_{PF}(Y_1 \text{ is odd at random}) = \frac{2}{3}$

(1 or more T-S in family of 5, both parents carriers) = (exactly 1 T-S) $\boxed{\text{or}}$ (exactly 2 T-S) $\boxed{\text{or}}$ . . . $\boxed{\text{or}}$ (exactly 5 T-S)

$$P(A \text{ or } B) \overset{?}{=} P(A) \, ? \, P(B)$$

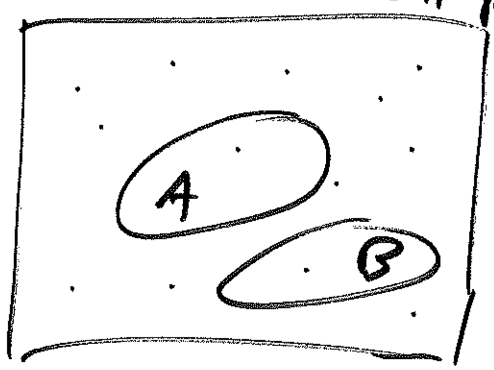
(1 or more) = $\boxed{\text{not}}$ (exactly 0 T-5)

$P(A) \overset{?}{=} P(\text{not } A)$

(exactly 0 T-5) = (not T-5 on 1st) $\boxed{\text{and}}$ (not T-5 on 2nd) $\boxed{\text{and}}$

... $\boxed{\text{and}}$ (not T-5 on 5th)

$P(A \text{ and } B) \overset{?}{=} P(A) \,?\, P(B)$



all possible outcomes Ω

$P(A \text{ or } B) =$

$P(A) + P(B)$ ✓

special case rule for or with no overlap

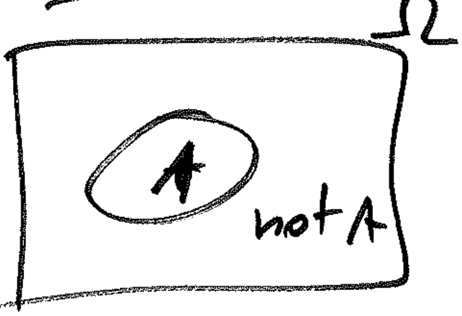
overlap
(A and B)

$P(A \text{ or } B) =$
$P(A) + P(B)$
$- P(A \text{ and } B)$

general rule for or

$(A, B \text{ no overlap}) \Leftrightarrow (A, B \text{ are mutually exclusive})$

$\Leftrightarrow \quad P(A \text{ and } B) = 0$



direct ~~indirect~~

$$\boxed{P(A) = 1 - P(\text{not } A)}$$

$$P(A) + P(\text{not } A) = 1$$
$$= 100\%$$

$P(A)$

set

$P(A)$

T/F statement

(proposition)

Stone's representation theorem