this time: 2 random variables at a time

next time: (DS ch. 3 sec 3.4) → 3.7

strong desire in math: generalize

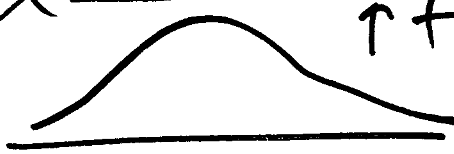
STAT 131
30 Apr 20
(lecture)

1

weird stuff

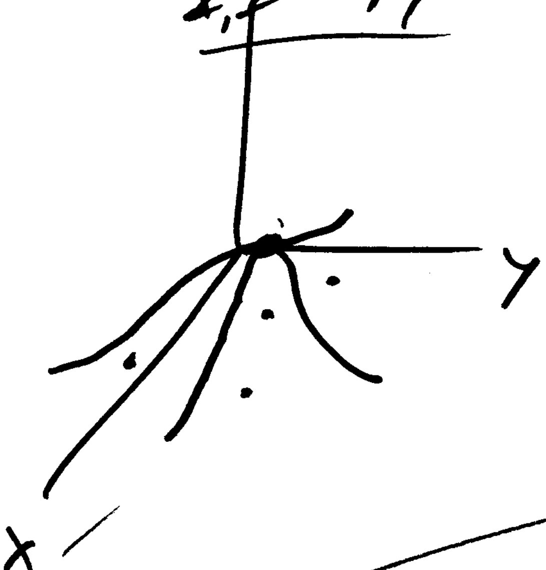
non-weird stuff

X continuous (univariate)



$f_X(x)$ PDF

x



$f_{X,Y}(x, y)$

normal (Gaussian) dist.

(bell curve)

unimodal

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

$$f_{X,Y}(x, y) = \begin{cases} c\, x^2 y & \text{for } 0 \le x^2 \le y \le 1 \\ 0 & \text{else} \end{cases}$$

(for some $c > 0$)

bivariate support set is $S_{X,Y} = \{(x,y): \underline{\underline{0 \le x^2 \le y \le 1}}\}$ (2)

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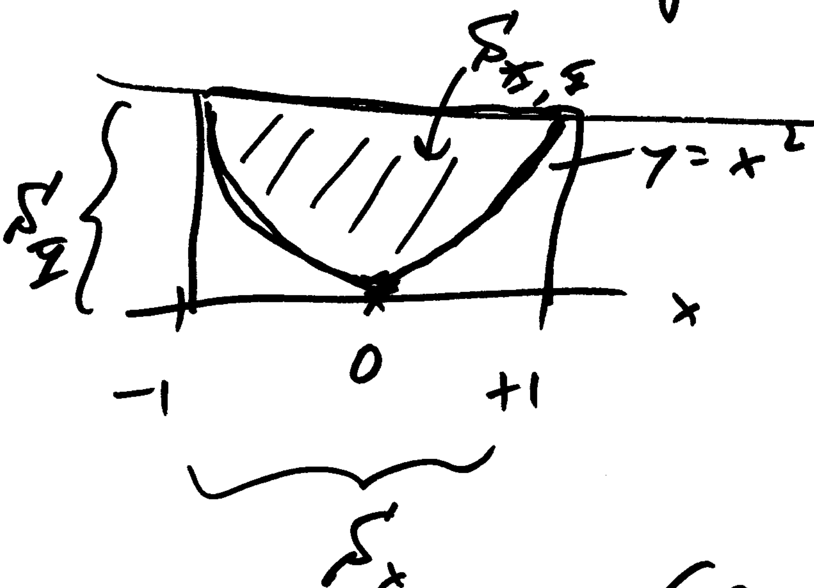
(1) work out univariate support for X and Y separately



(a) $0 \le x^2 \le 1$

$S_X = [-1, 1]$

(b) $S_Y = \{y: 0 \le y \le 1\}$

$(c > 0)$

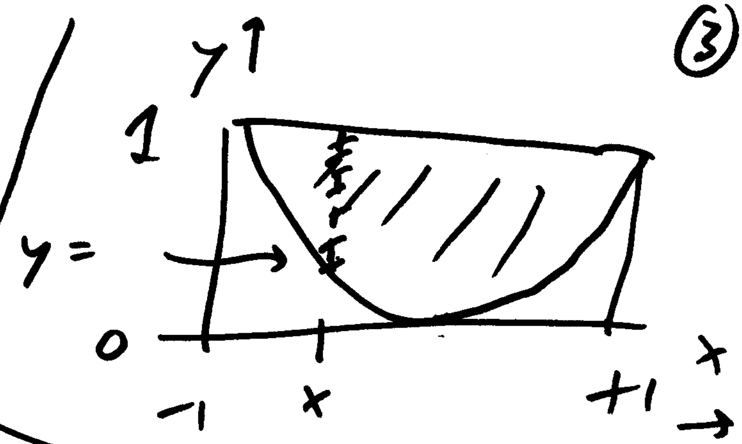
$$f_{X,Y}(x,y) = \begin{cases} c\,x^2 y & \text{for } (x,y) \in S_{X,Y} \\ 0 & \text{else} \end{cases}$$

$\underline{f_{X,Y}(x,y) \ge 0}$

$$\iint_{S_{X,Y}} f_{X,Y}(x,y)\, \underline{dx\, dy} = 1$$

$$= \iint_{S_{X,Y}} f_{X,Y}(x,y)\, dy\, dx$$

1. $\iint_{S_{X,Y}} c x^2 y \, dy \, dx$



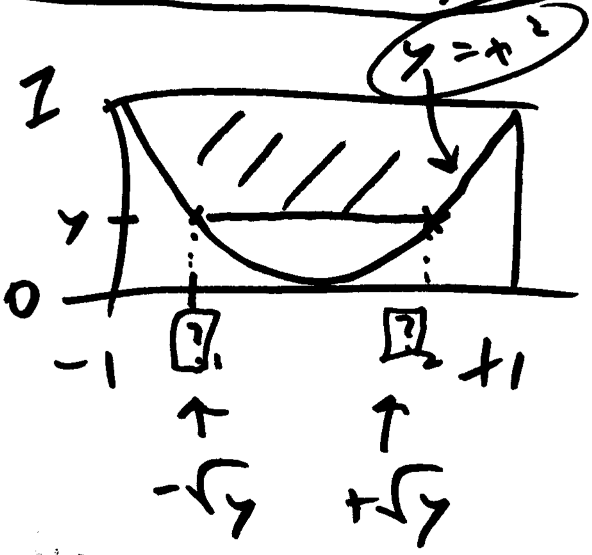
$$= \int_{-1}^{+1} \left[\int_{x^2}^{1} c x^2 y \, dy \right] dx$$

$$= \int_{-1}^{+1} \frac{1}{2} c x^2 (1 - x^4) \, dx$$

$$= \frac{4c}{21} = 1$$

$$C = \frac{21}{4}$$

2. $\iint_{S_{X,Y}} c x^2 y \, dx \, dy =$

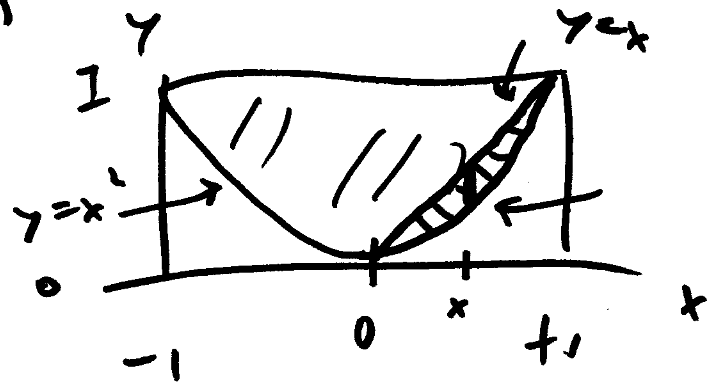


$$\int_{0}^{1} \left[\int_{-\sqrt{y}}^{+\sqrt{y}} c x^2 y \, dx \right] dy$$

$$(x = \pm\sqrt{y})$$

$$= \int_{0}^{1} \frac{2}{3} c y^{\frac{5}{2}} \, dy$$

$$= 4c/21$$



y = x

$$f_{X,Y}(x,y) = \begin{cases} \frac{21}{4}x^2 y & \text{for } (x,y)\in S_{X,Y} \\ 0 & \text{else} \end{cases}$$

$P(X \geq Y)$

$$= \int_0^{+1}\left[\int_{x^2}^{x} \frac{21}{4}x^2 y \, dy\right] dx = \frac{3}{20}$$

A, B true/false propositions

A, B independent iff $P(A \text{ and } B) = P(A)\,P(B)$

(X, Y) joint discrete distribution

$$f_{X,Y}(x,y) = P(X = x \overset{\text{and}}{,} Y = y)$$

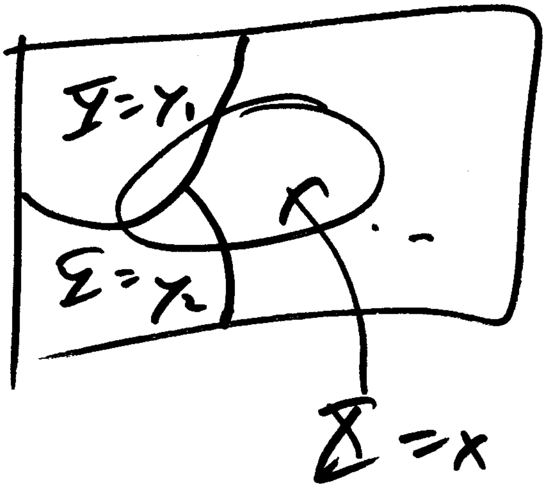
$$f_X(x) = P(X = x) =$$

$$\boxed{P(X = x, Y = y_1)}$$

$$+ P(X = x, Y = y_2)$$

$$+ \dots$$

$$\sum_{\text{all } y} f_{X,Y}(x, y)$$



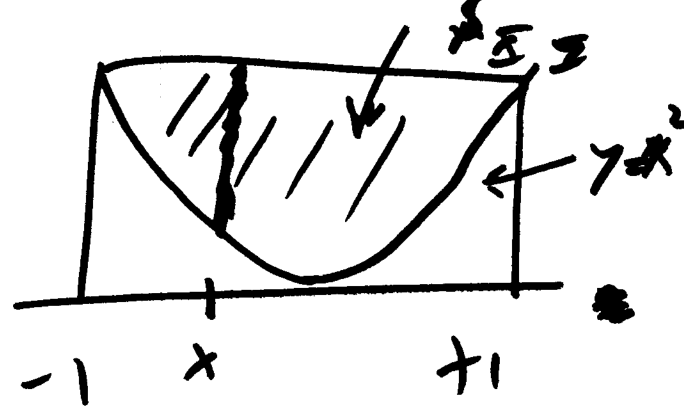
$$f_{X,Y}(x, y) = P(X = x \overset{\text{and}}{,} Y = y)$$

X, Y joint continuous pdf

$$f_X(x) = \int_{\text{all } y} f_{X,Y}(x, y)\, dy$$

$$f_{X,Y}(x,y) = \begin{cases} \frac{21}{4}x^2 y & \text{for } 0 \le x^2 \le y \le 1 \\ 0 & \text{else} \end{cases}$$

for $-1 \le x \le 1$



$$f_X(x) = \int_{x^2}^{1} f_{X,Y}(x,y)\,dy$$

$$= \int_{x^2}^{1} \frac{21}{4} x^2 y \, dy$$

$$= \frac{21}{8} x^2 (1 - x^4)$$