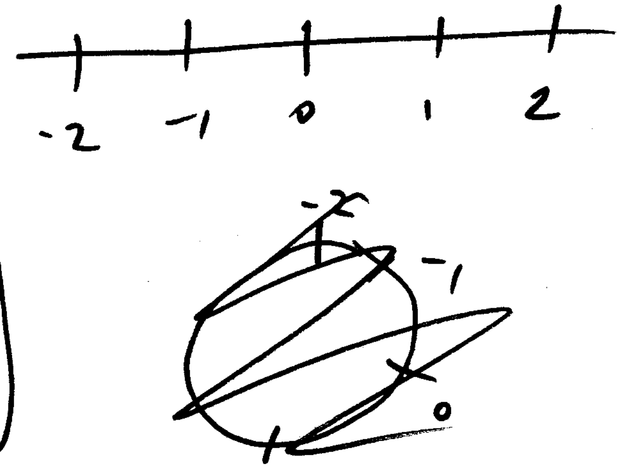
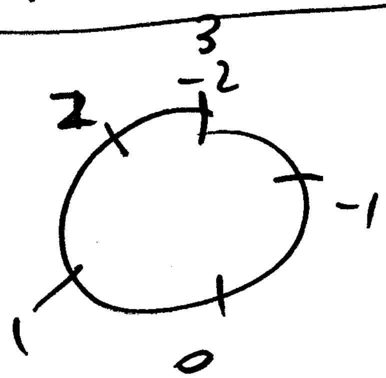
this time: Markov chains

next time: various catch-up topics

STAT 131
3 Jun 20

Catch-up Lecture





$$\lim_{n \to \infty} \underset{\sim}{v} P^n = ?$$

Suppose we could find a $\underset{\sim}{v}$ such that $\boxed{\underset{\sim}{v} P = \underset{\sim}{v}}$; then

$$\underset{\sim}{v} P^2 = (\underset{\sim}{v} P) P = \underset{\sim}{v} P = \underset{\sim}{v}$$

$$\underset{\sim}{v} P^3 = (\underset{\sim}{v} P) P^2 = \underset{\sim}{v} P^2 = \underset{\sim}{v}$$

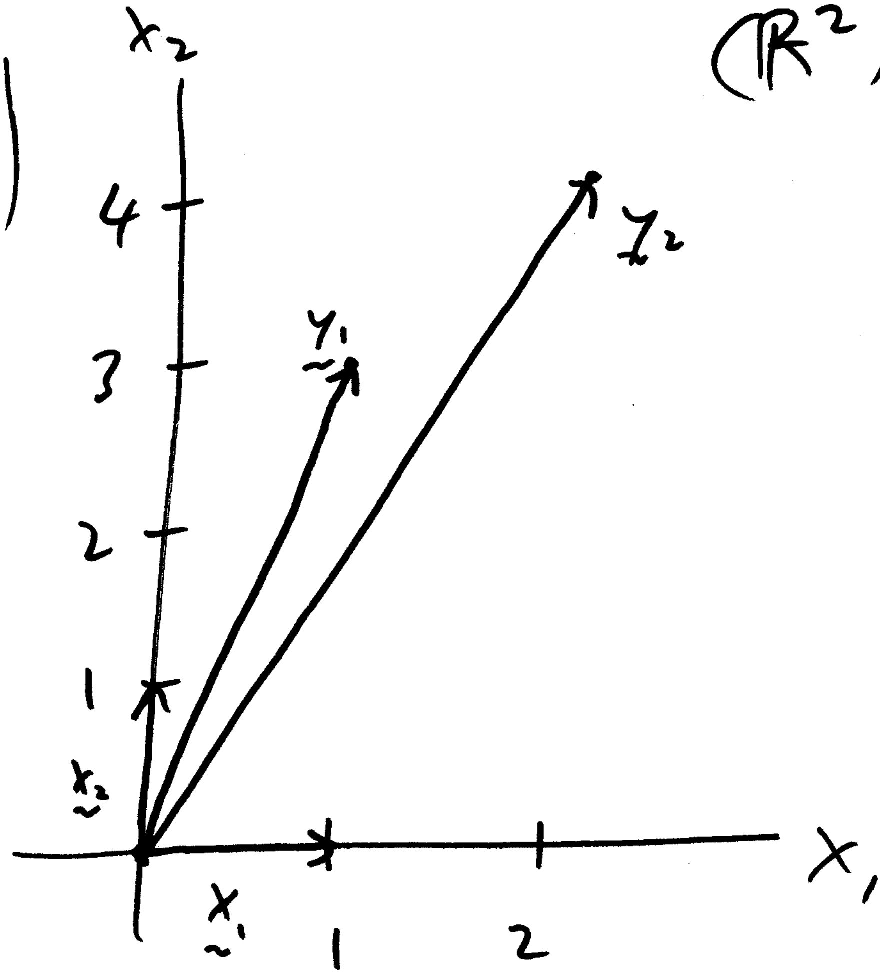
$$\lim_{n \to \infty} \underset{\sim}{v} P^n = \underset{\sim}{v}$$

k = 2 dimensions

linear algebra: eigen{values, vectors}

($\mathbb{R}^2$)

$$\underset{2\,2}{A} = \begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix} = \begin{pmatrix}a & b\\ c & d\end{pmatrix}$$



$$\cancel{= \begin{pmatrix} y_1 & y_2 \end{pmatrix}}$$

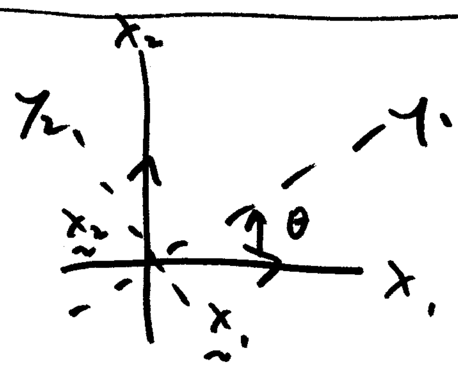
$$\underset{2\,2}{A}\;\underset{2\,1}{\underset{\sim}{x}} = \underset{\sim}{y}_1$$

$$\underset{2\,2}{\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}}\underset{2\,1}{\begin{pmatrix}1\\ 0\end{pmatrix}} = \begin{pmatrix}1\\ 3\end{pmatrix} = \underset{\sim}{y}_1$$

$$\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix} = \begin{pmatrix}2\\ 4\end{pmatrix} = \underset{\sim}{y}_2$$

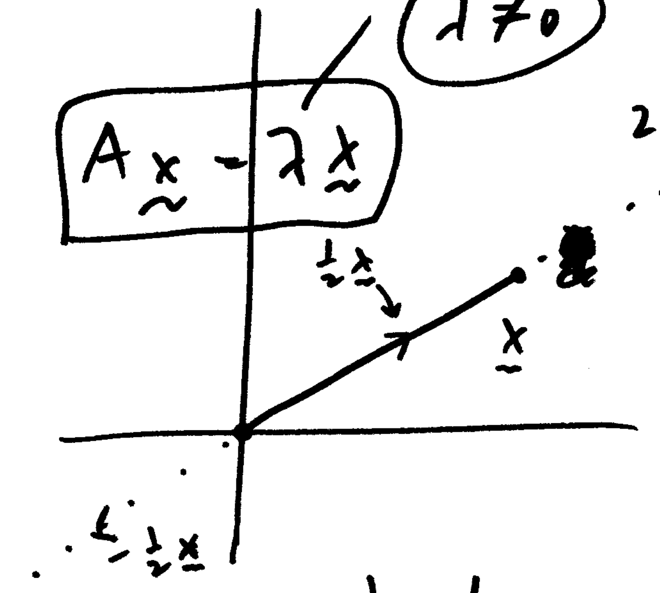
$A\,\underset{\sim}{x}$

rotates the basis vectors



Given a square matrix ${}_2A_2$, what

$\boxed{A\underset{\sim}{x} = \lambda \underset{\sim}{x}}$ $\lambda \neq 0$



if we could find a vector $\underset{\sim}{x}$ such that the action of A on $\underset{\sim}{x}$ involves <u>no</u> rotation at all, only compression or expansion or reflection ~~about~~ across the origin?

This would represent something fundamental about (<u>inherent in</u>) the matrix A

German: eigen

if ${}_2A_2 \cdot \underset{\sim}{X}_1 = \lambda_R \, {}_2\underset{\sim}{x}_1$, $\underset{\sim}{x}$ is a right eigenvector of A with eigenvalue λ_R

$\underset{1\quad 2}{\underset{\sim}{x}^T} A = \lambda_2 \underset{1\quad 2}{\underset{\sim}{x}^T} \rightarrow \underset{\sim}{x}^T$ is a <u>left</u>

<u>eigenvector with eigenvalue λ_2</u>