this time: normal distribution

next time: large random samples

read: ch. 6 in DS

STAT 131
29 May 20

A_2 approx.

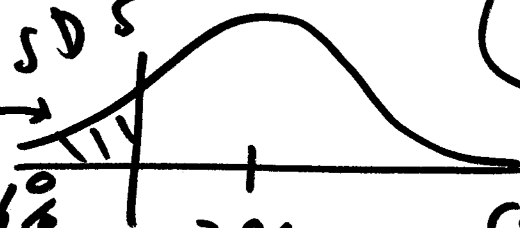
catch-up lecture

① A_1: (exact) 7.8%

SD 1 standard normal

0

? SD $s = 0.29$



5.6%

3.5

3.96 = mean $\bar{y}$

(cm) (y) (raw units)

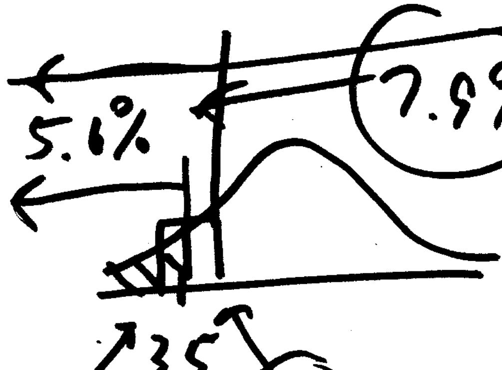
0

z (standard units axis)

need to convert 3.5 to standard units

$$-1.59 \doteq \frac{-0.46}{0.29} = \frac{3.5 - 3.96}{0.29}$$

$$z = \frac{y - \bar{y}}{s}$$



5.1%

7.9%

3.5

3.45

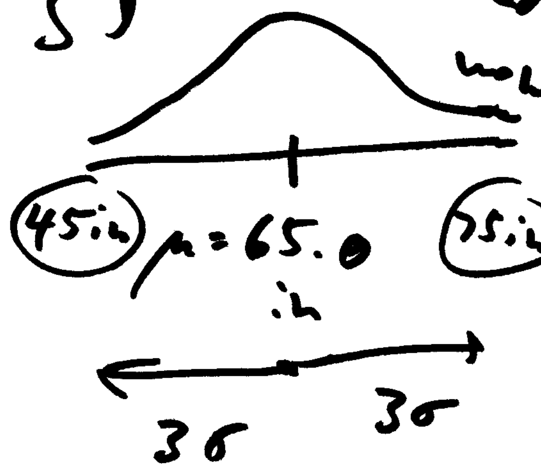
3.55

normal approximation with continuity correction

$$\frac{3.55 - 3.96}{0.29} = -1.41$$

SD $\sigma = 3.2$ in

height of adult U.S. women

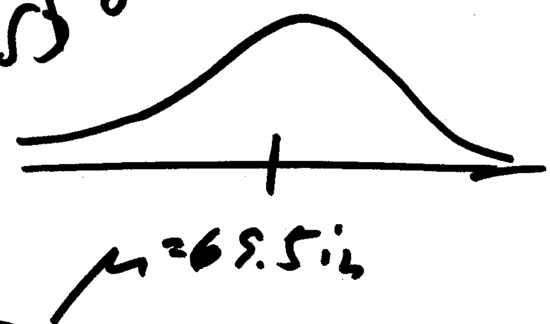


$\sigma = \cancel{1}$ (too small)

$\sigma = \cancel{5}$ (too big)

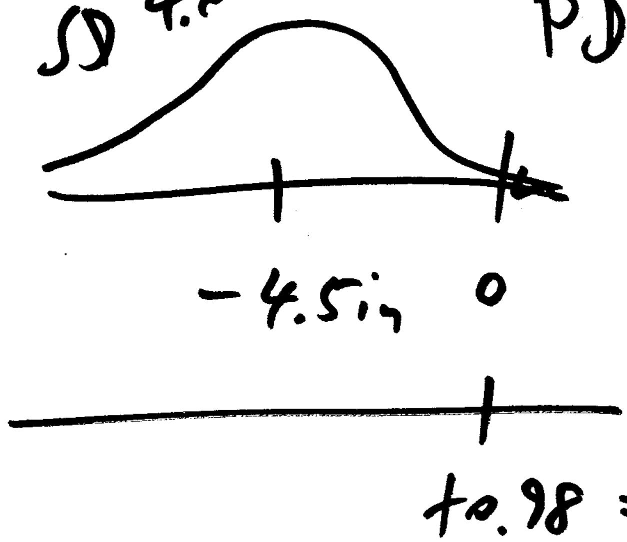
SD $\sigma = 3.3$ in

height of adult men (U.S.)



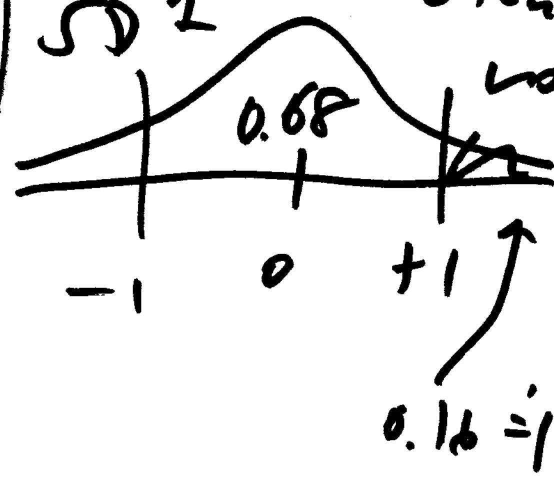
SD 4.6 in

PDF of D



SD 1

standard normal



money per person needed to get policy implemented

$T = 50M$

total amount of money needed $= \$\, T \cdot \bar{y}$

$$(X_i \mid \mu, \sigma^2) = X_i \overset{IID}{\sim} N(\mu, \sigma^2) \quad (i = 1, \ldots, n)$$

population

sample

$$\bar{X}_n \sim N\left(\mu, \frac{\sigma^2}{n}\right) ; \quad SD(\bar{X}_n) = \frac{\sigma}{\sqrt{n}}$$

$$E(\bar{X}_n) = E\left(\frac{1}{n} \cdot \sum_{i=1}^{n} X_i\right) = \frac{1}{n} E\left(\sum_{i=1}^{n} X_i\right)$$

$$= \frac{1}{n} \sum_{i=1}^{n} E(X_i) \overset{I\,ID}{=} \frac{1}{n}\left(\sum_{i=1}^{n} \mu\right) = \frac{n\mu}{n} = \mu$$

$$E(\bar{X}_n) = \mu \rightarrow \text{bias}(\bar{X}_n) = 0$$

$$V(\bar{X}_n) = V\left(\frac{1}{n} \cdot \sum_{i=1}^{n} X_i\right) = \frac{1}{n^2} V\left(\sum_{i=1}^{n} X_i\right)$$

$$\overset{I\,ID}{=} \frac{1}{n^2} \sum_{i=1}^{n} V(X_i) \overset{I\,ID}{=} \frac{1}{n^2} \sum_{i=1}^{n} \sigma^2 = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

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estimator

thing being estimated

$$\text{bias}(\hat{\theta}) = E(\hat{\theta}) - \theta$$

$$\text{bias}(\bar{X}_n) = E(\bar{X}_n) - \mu = 0$$

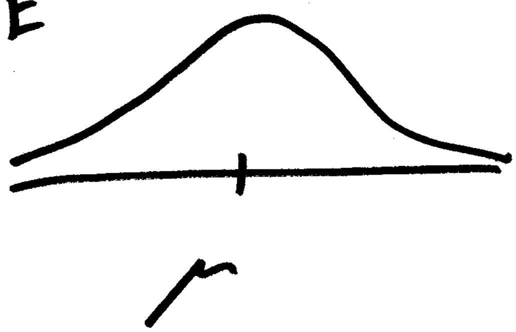
$SD(\hat{\theta}) \triangleq SE(\hat{\theta})$

estimator of θ

standard error of $\hat{\theta}$

$X_i \overset{IID}{\sim} N(\mu, \sigma^2)$ $(i = 1, \dots, n)$

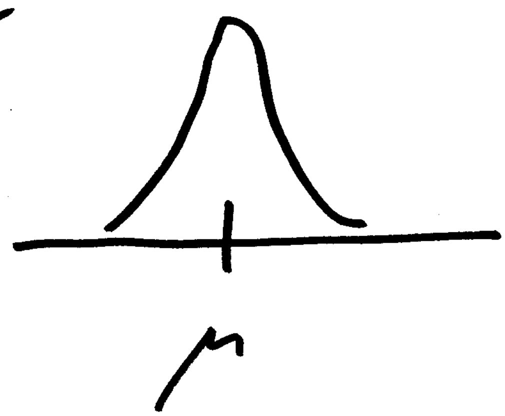
$SE = SD = \frac{\sigma}{\sqrt{1}} = \sigma$



PDF of $\bar{X}_n$, $n = 1$

this is the (weak) law of Large Numbers (WLLN)

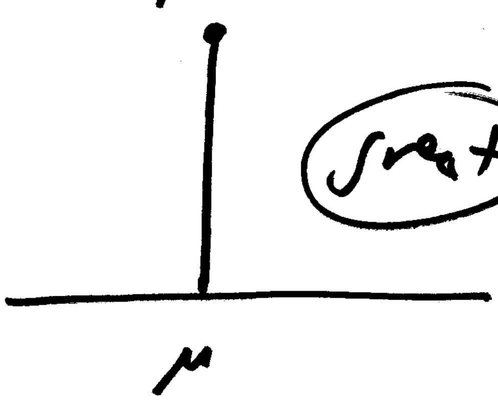
$SE = SD = \frac{\sigma}{\sqrt{10}}$



PDF of $\bar{X}_n$, $n = 10$

James Bernoulli (1705)

$X_i \sim$ Bernoulli(p)



great

PDF of $\bar{X}_n$, $n \to \infty$

asymptotics

population

all deer at UBSC on 1 Jun 2020

disease?

Is & os

N: 850

~~IID~~

sample

the observed deer

disease?

$X_1, \dots, X_n$ | Is & os | $x_1, \dots, x_n$ | n

mean $\bar{X}_n$ | $\bar{x}_n$

mean $\theta = ?$

or μ