This normal time: distribution next large time: samples

1. We take a break this Fri, Sat & Sun, catch-up lectures in week 1.
2. We push through, will catch-up lecturing & extra office hours every day from now until Fri 5 Jun, 20.

Both options: extra office hours at least from next Mon every day until Sun 14 Jun.

PDF \( \sim \mathcal{N}(\mu, \sigma^2) \) normal distribution (Gaussian)

- \( \text{E}(\bar{X}) = \mu_{\bar{X}} \)
- \( \text{V}(\bar{X}) = \frac{\sigma^2}{n} \)
- \( \text{SD}(\bar{X}) = \frac{\sigma}{\sqrt{n}} \)

First to find PDF formula:

A de Moivre (1718):

\[
f_{\bar{X}}(x | \mu_{\bar{X}}, \sigma^2) = \frac{1}{\sigma_{\bar{X}} \sqrt{2\pi}} \exp \left( -\frac{1}{2} \left( \frac{x - \mu_{\bar{X}}}{\sigma_{\bar{X}}} \right)^2 \right)
\]
\[ E(\bar{X}) = E(a\bar{X} + b) = aE(\bar{X}) + b \]
\[ V(\bar{X}) = V(a\bar{X} + b) = a^2 V(\bar{X}) \]

Support of \( \bar{X} \sim N(\mu_X, \sigma_X^2) \) is \((-\infty, +\infty)\).

\[ \int e^{-t^2} dt \text{ has no antiderivative in closed form.} \]
$F_{\mathcal{N}}(x)$ has to be approximated with numerical integration. Can make a table.

$\Phi \left( \frac{x - \mu}{\sigma} \right)$, $\sigma \in \mathbb{R}$, $\mu \in \mathbb{R}$.

Standard normal distribution.

$F_{\mathcal{N}}(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp \left[ -\frac{t^2}{2} \right] \, dt$

$\mathcal{N}(0,1)$

$\Phi(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{(x - \mu)^2}{2\sigma^2} \right]$.

David Freedman

$Z = \frac{Y - \mu_0}{\sigma_0}$

units (z)

unit axis (z)

$z = \frac{Y - \mu_0}{\sigma_0}$

standard
To get $\kappa$, convert $g$ to states.
PDF of \( X \sim N(0, 1) \) \( P(X \leq x) = \Phi(x) \)

\[ \Phi(x) \]

\[ 1 - \Phi(x) = \Phi(-x) \quad \text{for all} \]

\[ \Phi(-x) + \Phi(x) = 1 \]

PDF of \( X \sim N(\mu, 1) \) \( P(X \geq x) = \Phi(-x) \)

PDF of \( X \sim N(\mu, 1) \) \( P(X < x) = \Phi(x) \)

\[ P(-x \leq X \leq x) = \Phi(x) - \Phi(-x) \]

\( N(0, 1) \):

1. left-tailed + \( \Phi \)
2. symmetric around 0
$X \sim N(0,1)$

$\text{CDF } F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt$

$\text{PDF } \phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

$I(0) = \frac{1}{2}$

$I(x) = \frac{1}{2} \text{erfc} \left( \frac{x}{\sqrt{2}} \right)$

$I(x) = \text{CDF} \left[ \text{normal}(0,1), x \right]$ from Alph
\[ I^{-1}(p) \sim N(0,1) \iff p = I(x_p) \]

Inverse CDF of \( N(0,1) \) distribution.

\[ I^{-1}(p) \iff \text{inverseCDF}[\text{normal}(0,1), p] \]

CDF of \( N(0,1) \) function.