STAT 131

28 May 20

(lecture)

1

this time: normal distribution

next time: large random samples

① we take a break this Fri, Sat & Sun; catch-up lectures in week 11

② we push through, will catch-up lecture & extra office hours every day from now until Fri 5 Jun 20

both options: extra office hours at least from next Mon every day until Sun 14 Jun

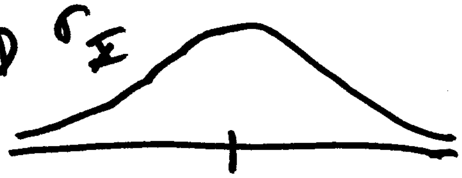
PDF r.v. X

normal (K. Pearson) distribution

(Gaussian)

first to find PDF formula: A de Moivre (1710)

SD σ_X



μ_X

① $E(X) = \mu_X$

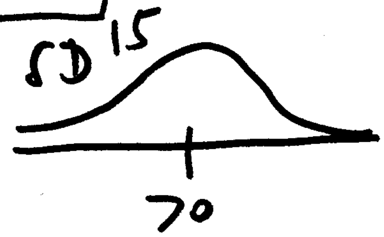
② $V(X) = \sigma_X^2$, $SD(X) = \sigma_X$

$(X \mid \mu_X, \sigma_X^2) \sim N(\mu_X, \sigma_X^2)$

$$f_X(x \mid \mu_X, \sigma_X^2) = \frac{1}{\sigma_X \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu_X}{\sigma_X}\right)^2\right]$$

PDF of X

add 20 $\longrightarrow$ ex. $\boxed{a=1,\ b=20}$



$$Y = aX + b \quad (a \neq 0)$$

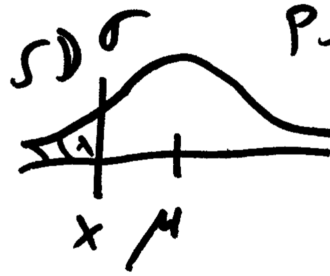
$$X \sim N(\mu_X, \sigma_X^2)$$

$$Y \sim N(a\mu_X + b,\ a^2\sigma_X^2)$$

$$E(Y) = E(aX + \underline{\underline{b}})$$
$$= E(aX) + b$$
$$= aE(X) + b$$

$$V(Y) = V(aX + b)$$
$$= V(aX)$$
$$= a^2 V(X)$$

PDF of X

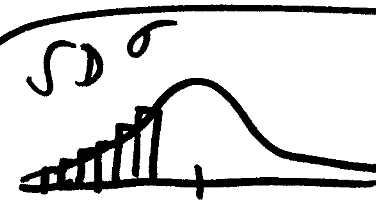


$$F_X(x) = \underline{P(X \le x)}$$
$$= \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\left(\frac{t-\mu}{\sigma}\right)^2\right] dt$$

Support of $X \sim N(\cdot,\cdot) = (-\infty, +\infty)$

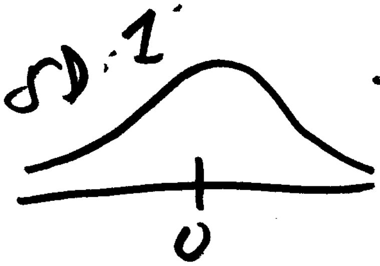
$\int e^{-t^2} dt \overset{?}{=}$ has no antiderivative in closed form

$F_X(x)$ has to be approximated with numerical integration; can make a table



DS p. 861

$\underline{\mu \in \mathbb{R}}$, $\sigma \in \cancel{[0, \infty)}$ $\underline{(0, \infty)}$



standard normal distribution

$$F_X(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{t^2}{2}\right] dt \triangleq \Phi(x)$$

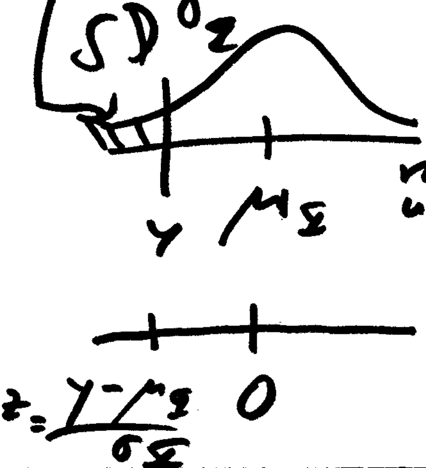
$\boxed{X \sim N(0,1)}$

capital phi

$\cancel{\Phi(z)}$

$(X \mid \mu_X, \sigma_X^2) \sim N(\mu_X, \sigma_X^2)$

David Freedman



raw units (X)

$P(X \le y \mid \mu_X, \sigma_X^2) = ?$

standard units axis (z)

④

to get ?, convert Y to standard units:

$$X = \frac{Y - \mu_Y}{\sigma_Y}$$

$E(X) = 0$

$V(X) = 1 = SD(X)$

$(Y \mid \mu_Y, \sigma_Y^2) \sim N(\mu_Y, \sigma_Y^2)$

$N(0, 1)$

$$F_Y(y \mid \mu_Y, \sigma_Y^2) =$$

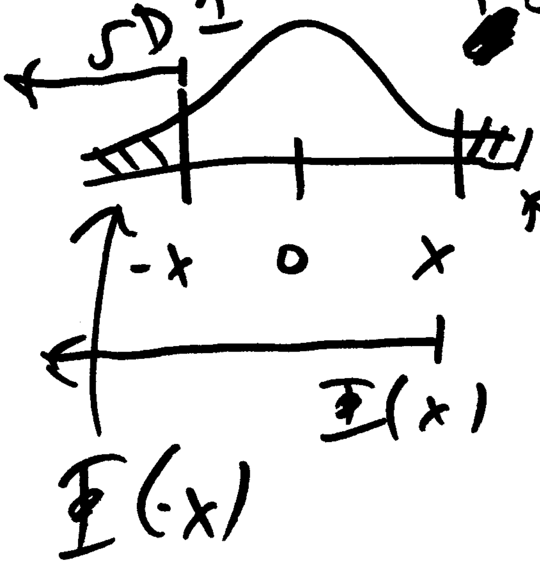
$$P(\underline{Y \le y} \mid \mu_Y, \sigma_Y^2)$$

$\sigma_Y > 0$

$$= P\left(\frac{Y - \mu_Y}{\sigma_Y} \le \frac{y - \mu_Y}{\sigma_Y} \,\middle|\, \mu_Y, \sigma_Y^2 \right)$$

$$= P\left(X \le \frac{y - \mu_Y}{\sigma_Y} \,\middle|\, \mu_Y, \sigma_Y^2 \right)$$

$$= \Phi\left(\frac{y - \mu_Y}{\sigma_Y} \right) ✓$$



PDF of

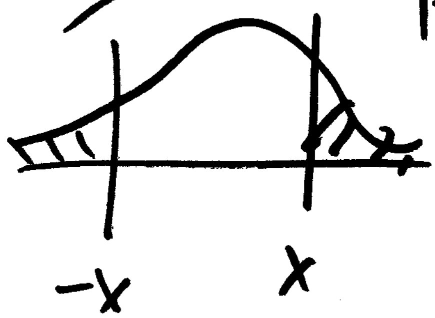
$\Phi \sim N(0,1)$ $\quad P(\Phi \le x) = \Phi(x)$

$1 - \Phi(x) = \Phi(-x)$ so

for all x

$$\Phi(-x) + \Phi(x) = 1$$

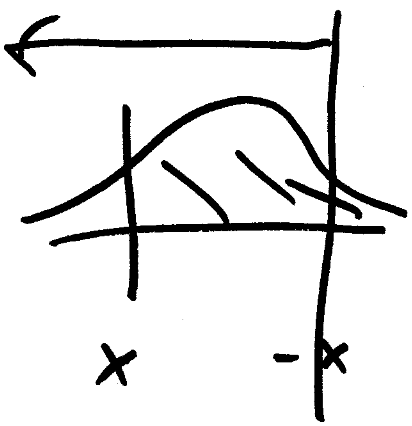
for all



PDF of

$\Phi \sim N(0,1)$ $\quad P(\Phi \ge x) =$

$\Phi(-x)$



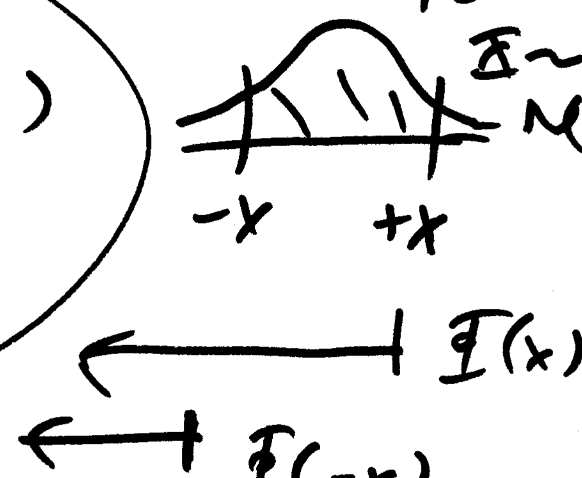
PDF of

$\Phi \sim N(0,1)$

$x < 0$

PDF of

$\Phi \sim N(0,1)$

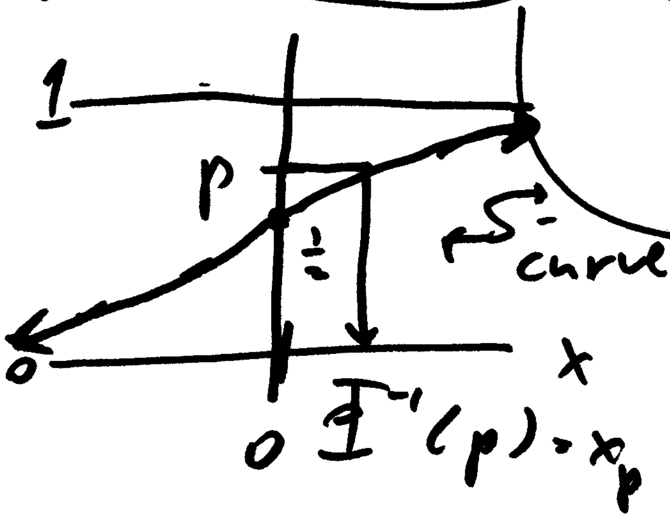


$$P(-x \le \Phi \le x) = \Phi(x) - \Phi(-x)$$

$N(0,1)$: ① integrates to 1
② symmetric around 0

$X \sim N(0,1)$

CDF $\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} \, dt$



PDF $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}$

$\boxed{\Phi(0) = \frac{1}{2}}$

Φ is strictly increasing from $-\infty$ to $+\infty$

$$\Phi(x) = \frac{1}{2} \operatorname{erfc}\left(-\frac{x}{\sqrt{2}}\right)$$

physics: error function

$\Phi(x)$ = CDF[normal(0,1), x]

wolfram Alpha

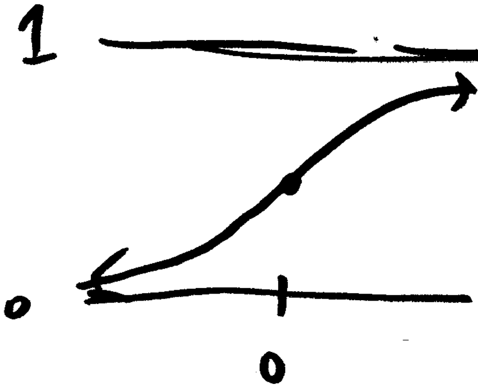
$$\Phi^{-1}(p) = x_p \longleftrightarrow p = \Phi(x_p)$$

inverse CDF of $N(0,1)$ dist

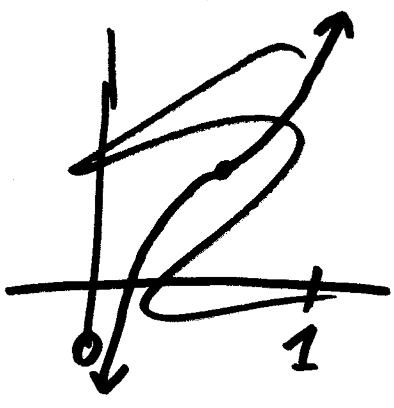
quantile function

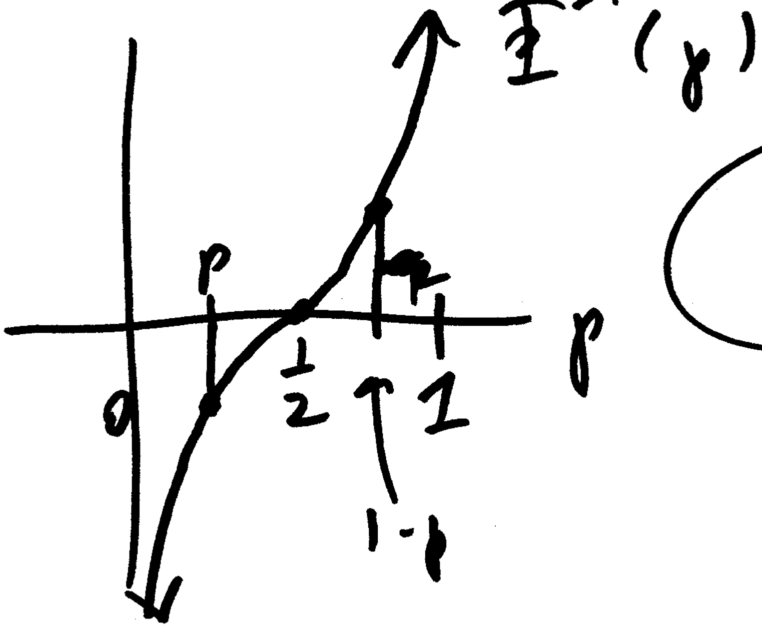
$\boxed{\text{Wa}}$ $\Phi^{-1}(p) \longleftrightarrow$

inverseCDF[normal(0,1), p]



CDF $\Phi(x)$ of $N(0,1)$





$$\Phi^{-1}(p) = -\Phi^{-1}(1-p)$$