this time: covariance, correlation;

next time: moments, conditional expectation & variance

DS ch. 4

STAT 131
21 May 20

(lecture)

1

1^{ST} central moment of X =

$$E(X-\mu)^1 = E(X-\mu)$$

$$= E(X) - E(\mu) \leftarrow \text{constant}$$

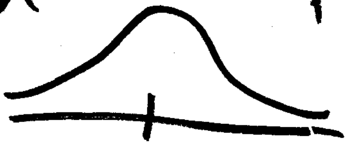
$$= E(X) - \mu$$

$$= E(X) - E(X) = 0$$

$SD(X) = \sqrt{V(X)} = \sqrt{2^{nd} \text{ central moment}}$

3rd central moment = 0

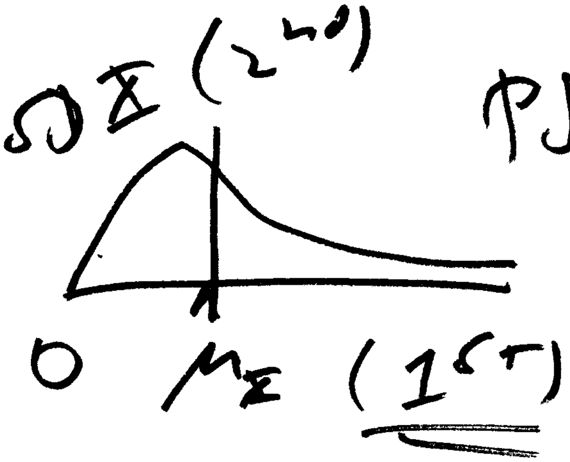
PDF of X



symmetric

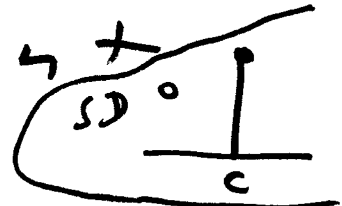
skewness moment

$\mu_X \leftarrow$ relates to 1^{ST} central moment



PDF of X

asymmetric → positive skewness ← 3rd central moment ✓



converting to standard units (assume $\sigma_X > 0$)

$$X \longrightarrow Z = \frac{X - \mu_X}{\sigma_X}$$

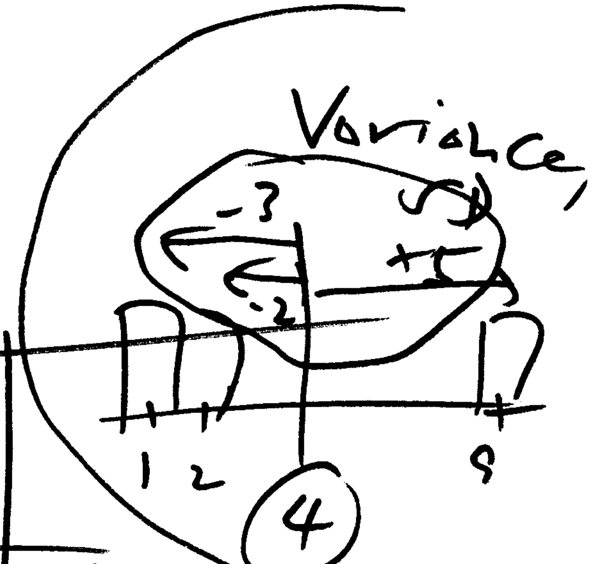
σ_X ← SD of X

Quantity	mean (1st)	SD (2nd)
X	μ_X	σ_X
$Z = \frac{X - \mu_X}{\sigma_X}$	0	1

$$E(Z) = E\left(\frac{X - \mu_X}{\sigma_X}\right) = \frac{1}{\sigma_X} E(X - \mu_X) = 0$$

$$SD(Z_X) = SD\left(\frac{X-\mu_X}{\sigma_X}\right) = \frac{SD(X-\mu_X)}{\sigma_X}$$

$$= \frac{SD(X)}{\sigma_X} = 1$$



Summary	what it measures	moment
mean	center	1
(Variance) SD	spread	2 (central)
skewness	shape_1	3 (standardized)
kurtosis	shape_2: tail weight (how much prob. is way away from middle)	4

$$e^{x} = \frac{1}{0!} + \frac{x}{1!} + \frac{x^2}{2} + \frac{x^3}{6} + \cdots$$

$x \in \mathbb{R}$

taylor series around $x = 0$

$2 = 2!$ $\quad$ $6 = 3!$

pick some number t not far from 0 but not 0

$$e^{tx} = 1 + tx + \frac{(tx)^2}{2!} + \frac{(tx)^3}{3!} + \cdots$$

$$e^{tX} = 1 + tX + \frac{t^2X^2}{2!} + \frac{t^3X^3}{3!} + \cdots$$

conjecture

$$E\left(e^{tX}\right) \overset{?}{=} 1 + E(tX) + E\left(\frac{t^2X^2}{2!}\right) + E\left(\frac{t^3X^3}{3!}\right) + \cdots$$

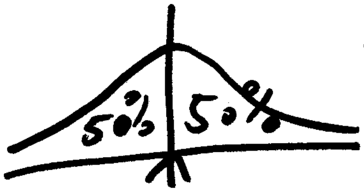
$$E(e^{tX}) = 1 + t\boxed{E(X)} + \frac{t^2}{2!}\boxed{E(X^2)} + \frac{t^3}{3!}\boxed{E(X^3)} + \dots$$

$$\frac{d}{dt}E(e^{tX}) = E(X) + \frac{2t}{2!}E(X^2) + \frac{3t^2}{3!}E(X^3) + \dots$$

$$\left(\frac{d}{dt}E(e^{tX})\right)_{t=0} = E(X)$$

$$\frac{d^2}{dt^2}E(e^{tX}) = E(X^2) + \frac{3!\,t}{3!}E(X^3) + \dots$$

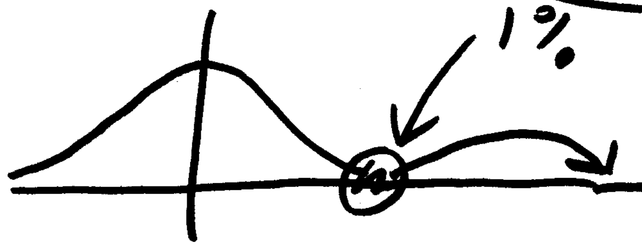
$$\left[\frac{d}{dt^2}E(e^{tX})\right]_{t=0} = E(X^2)$$



PDF of X

μ_X (mean)
= median
= mode
= pt. of symmetry

(yearly income) PDF of X (house prices)



mean ↑
median unchanged

(sell your house) → quote mean

(buying a house) → quote median