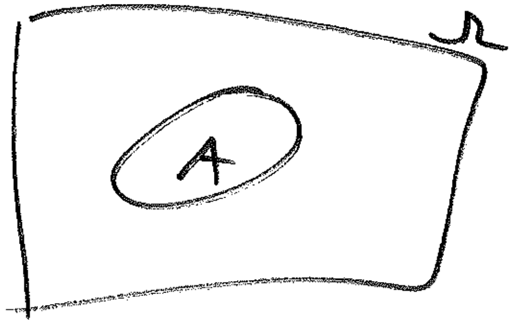
This time: basic probability rules

next time: foundations

read: DS ch. 1

STAT 131
2 Apr 20
lecture 1



$$0 \le P(A) \le 1$$

0% 100%

$P(A \text{ and } B) = ?$

$P(A) ? P(B)$

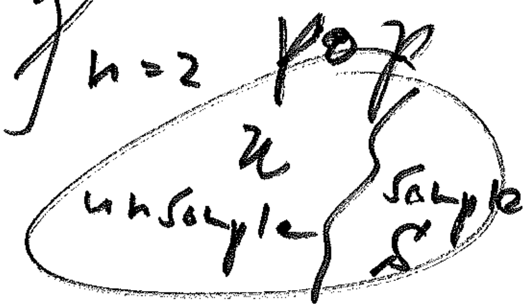
Case I

pop. sample (data)

random variables outcomes

at random with replacement

$N = 3$
$\begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$ at random $\rightarrow$ $\begin{matrix} X_1 \\ X_2 \end{matrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ $n = 2$



goal of sampling from pop.: try to make <u>sample</u> S & <u>unsample</u> U as <u>similar</u> as possible in all <u>relevant ways</u>

how achieve goal: choose S at random (2)

(at random with repl.) ⟷ IID sampling

independent identically distributed

(at random without replacement) ⟷ simple random sampling (SRS)

IID vs. SRS

math easier

more information

($n = 1$) IID = SRS

($n = N$) IID still has uncertainty but SRS has ~~no~~ uncertainty

$\underline{n} << N \leftrightarrow IID \doteq SRS$

"is a lot smaller than"

is approximately equal to

Case I IID

pop

sample

$N=3 \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} \xrightarrow{IID} \begin{array}{c} Y_1 \\ Y_2 \end{array} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \; n=2$

$P(Y_1 = 7$ and $Y_2 = 7) = ?$

ELM? yes.

2nd draw (Y_2)

1st draw (Y_1)

	1	4	7
1	(1, 1)	(1, 4)	(1, 7)
4	(4, 1)	(4, 4)	(4, 7)
7	(7, 1)	(7, 4)	(7, 7)

$P_{IID}(Y_1 = 7$ and $Y_2 = 7) = \dfrac{1}{9}$

$\frac{1}{3} = \frac{3}{9}$ $\qquad \frac{1}{3} = \frac{3}{9}$

$P_{IID}(Y_1 = 7) \cdot P_{IID}(Y_2 = 7)$

Case I
working theory

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

pop sample

Case II: SRS

$$N=3 \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix} \xrightarrow{SRS} \begin{array}{c} Y_1 \\ Y_2 \end{array} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \quad n=2$$

$$P_{SRS}(Y_1 = 7 \text{ and } Y_2 = 7) = 0$$

$$P_{SRS}(Y_1 = 7) = \frac{1}{3} \qquad P_{SRS}(Y_2 = 7) = \frac{2}{6} = \frac{1}{3}$$

2nd draw (Y_2)

1st draw (Y_1)	1	4	7
1	~~(1,1)~~	(1,4)	(1,7)
4	(4,1)	~~(4,4)~~	(4,7)
7	(7,1)	(7,4)	~~(7,7)~~

ELM? yes

$$P_{SRS}(Y_1 = 7 \text{ and } Y_2 = 7) = 0 \neq \underbrace{P_{SRS}(Y_1 = 7)}_{1/3} \cdot \underbrace{P_{SRS}(Y_2 = 7)}_{1/3}$$

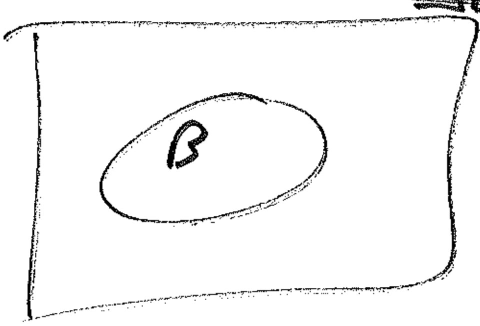
def. | conditional probability (5)

Abraham de Moivre (1710)

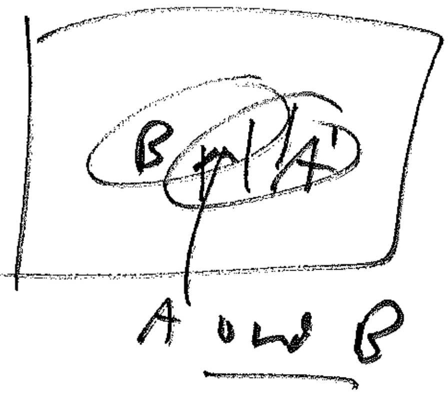
$$P(B \mid A) = \begin{cases} \dfrac{P(B \text{ and } A)}{P(A)} & \text{if } P(A) > 0 \\ \text{undefined} & P(A) = 0 \end{cases}$$

"given"

Thomas Bayes (1761)



$$P(B) = \frac{B}{\boxed{1}}$$



$$P(B \mid A) = \frac{A \text{ and } B}{A}$$

Suppose that $P(A) > 0$

(chain rule for prob)

$$P(B \text{ and } A) = P(A \text{ and } B)$$

$$= P(A) \cdot P(B \mid A)$$

$$= P(B) \cdot P(A \mid B)$$

6

$$P(A \mid B) = \begin{cases} \dfrac{P(A \text{ and } B)}{P(B)} & \text{for } P(B) > 0 \\ \text{undefined} & \text{else} \end{cases}$$

if $P(B) > 0$

$P(A \text{ and } B) = P(B) \cdot P(A \mid B)$

$$P_{SRS}(\underline{\Sigma_1 = 7} \text{ and } \underline{\Sigma_2 = 7})$$

$$= P_{SRS}(\Sigma_1 = 7) \cdot P_{SRS}(\Sigma_2 = 7 \mid \underline{\underline{\Sigma_1 = 7}})$$

$$= \left(\frac{1}{3}\right) \cdot 0 = 0 \checkmark$$

AND $P(A \text{ and } B) = P(A) \cdot P(B|A)$

general rule for AND

$= P(B) \cdot P(A|B)$

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in special case in which A, B are <u>independent</u>

$P(A \text{ and } B) = P(A) \cdot P(B)$ ✓

$= P(B) \cdot P(A)$ ✓

Bayesian definition of independence

$\underline{\underline{A}}, \underline{\underline{B}}$ independent if information about A does not change chances for B, & vice versa frequentist definition of independence

A, B independent iff $p(A \text{ and } B) = p(A) \cdot p(B)$

$p($1 or more Ts in family of 5 children both parents carriers$)$

A

$= p($exactly 1 Ts or ... or exactly 5 Ts$)$

mut. excl. $= p($exactly 1 Ts$) + p($exactly 2$) + \ldots + p($exactly 5$)$

$= 1 - p(\text{not } A)$

$= 1 - p($exactly 0 T-S$)$

9

$$= 1 - P\left(\begin{matrix}\text{not TS}\\ \text{on}\\ \underline{\underline{1^{st}}}\end{matrix} \text{ and } \begin{matrix}\text{not TS}\\ \text{on}\\ \underline{\underline{2^{nd}}}\end{matrix} \text{ and} \cdot \text{and } \begin{matrix}\text{not}\\ \text{TS}\\ \text{on}\\ 5^{th}\end{matrix}\right)$$

I (indep)

$$= 1 - \underline{\underline{P\left(\begin{matrix}\text{not TS}\\ \text{on}\\ 1^{st}\end{matrix}\right)}} \cdot P\left(\begin{matrix}\text{not}\\ \text{TS}\\ \text{on}\\ 2^{nd}\end{matrix}\right) \cdot \ldots P\left(\begin{matrix}\text{not}\\ \text{TS}\\ \text{on}\\ 5^{th}\end{matrix}\right)$$

II iden. dist.

$$= 1 - \left(1 - \underset{=}{\frac{1}{4}}\right) \cdot \left(1 - \underset{=}{\frac{1}{4}}\right) \cdots \left(1 - \underset{=}{\frac{1}{4}}\right)$$

$$P(A) = 1 - \left(1 - \frac{1}{4}\right)^5 \doteq 76\%$$

$$= \frac{781}{1024}$$

P(one or more bad things)

$$= 1 - (1-p)^n$$

n ← # of thngs

p ← P(bad thing on any single try)