

STAT 131
16 Apr 20

this time: Bayes's theorem
next time: random variables

for A any event (true/false proposition),
let $P(A) = p_A$; then

the odds ratio in favor of A being true is defined to be $o_A = \frac{P(A)}{P(\text{not } A)} = \frac{p_A}{1-p_A}$
odds ratio in terms of probability

You can readily solve backwards to get $p_A = \frac{o_A}{1+o_A}$ = probability in terms of odds ratio

p_A	o_A
0.01	$\frac{1}{99}$
0.1	$\frac{1}{9}$
0.5	$\frac{1}{1}$
0.9	$\frac{9}{1}$
0.99	$\frac{99}{1}$

odds ratios are used by people juggling on things like horse races, but they're also useful in medical research.

$$P(C|E) = \frac{P(C \text{ and } E)}{P(E)}$$

$$P(E|C) = \frac{P(E \text{ and } C)}{P(C)}$$

$$P(E) > 0$$

$$P(C) > 0$$

$$P(C|E) = \frac{P(C \text{ and } E)}{P(E)}$$

$$P(C \text{ and } E) = P(E) P(C|E)$$

$$P(E|C) = \frac{P(E \text{ and } C)}{P(C)}$$

$$P(E \text{ and } C) = P(C) P(E|C)$$

$$P(E) P(C|E) = P(C) P(E|C)$$

$$P(C|E) = \frac{P(C)P(E|C)}{P(E)}$$

(3)

Bayes's Theorem for single events (T/F propositions)

Case Study: Screening for COVID-19

truth: \textcircled{C} really is covid-19 positive

$\textcircled{not C}$ really not

what Collex test says:

test \oplus thinks covid-19 positive

test \ominus negative

truth	test says
$\textcircled{not C}$	\ominus
\textcircled{C}	\oplus

1,000,000

test says

	\textcircled{C}	$\textcircled{not C}$
\oplus		
\ominus		

1000000

have to know 3 things: $P(C) = 0.01$ (4)

① prevalence of COVID-19 in

U.S. pop. : Case 1: $\frac{64,116}{328,200,000} = \underline{\underline{0.2\%}}$

Case 2: 1.0%

② $P(+ | C) = \underline{\underline{93.8\%}}$
 = sensitivity of cellex test

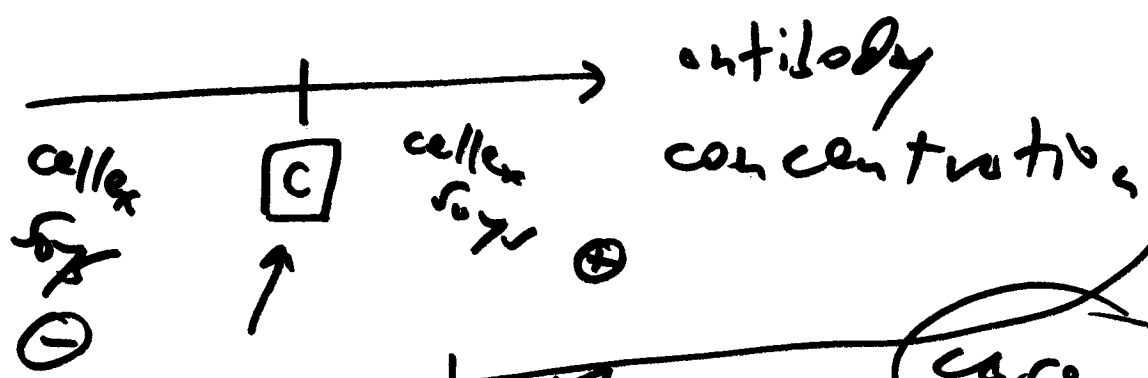
③ $P(- | \text{not } C) = \text{specificity of } \uparrow$
 = 95.6%

method I

	<u>C</u>	<u>not C</u>	
<u>cellex test says</u> <u>+</u>	9,380	43,560	52,940
<u>cellex test says</u> <u>-</u>	620	946,440	947,060
	10,000	990,000	1,000,000

false positive: 43,560
 false negative: 620
 culprit: Case 2
 $P(C) = 1\% = \frac{10,000}{1,000,000}$
 $P(\text{not } C | -) = \frac{946,440}{947,060} = 0.9993$

$P(C | +) = \frac{9,380}{52,940} = 0.177 = \underline{\underline{18\%(!)}}$



case 2

	(C)	note	
cellx (+)	1,876	43,912	45,788
test (-)	124	954,088	954,212
	2000	99,800	1,000,000

$$P(C) = 0.002$$

$$P(C | +) = \frac{1,876}{45,788} = 0.04 (!)$$

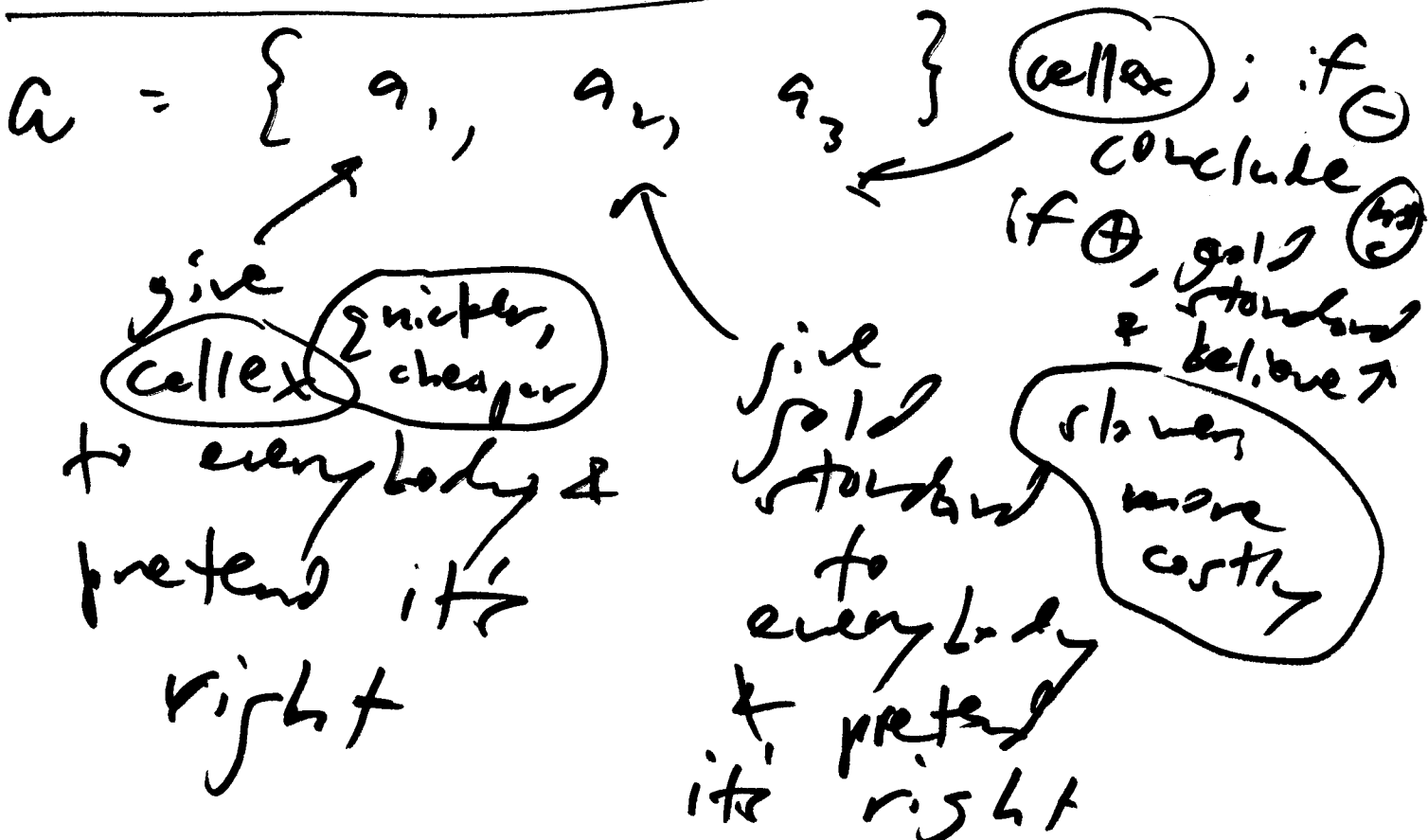
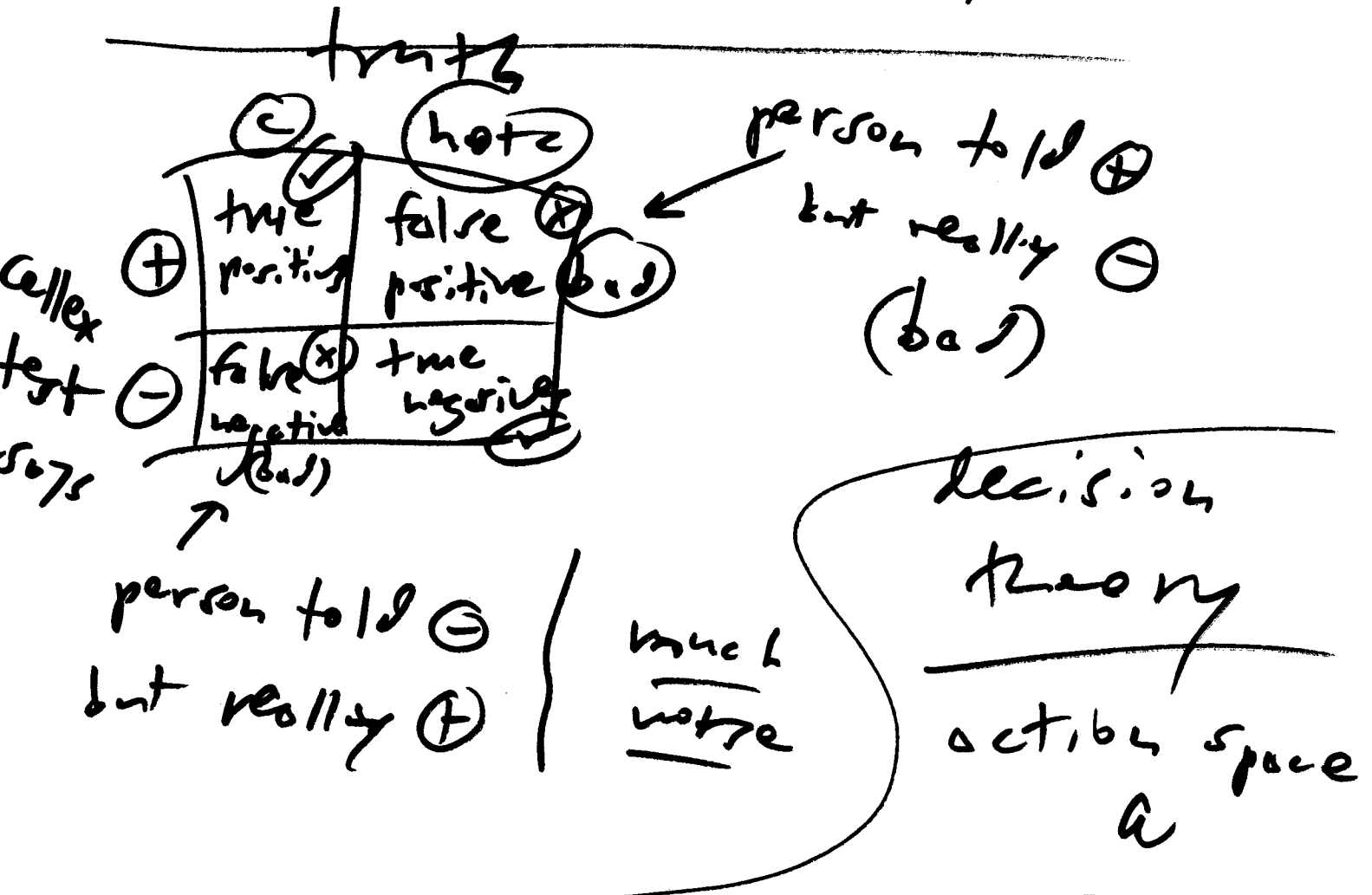
$$P(\text{not } C | -) = \frac{954,088}{954,212} = .99987$$

case	$P(C +)$	$P(\text{not } C -)$	prevalence
1	4%	99.99%	0.2%
2	18%	99.93%	1.0%

cellx test almost perfect when (-)

but it's ~~diff~~ when it says ⊕

⑥



know:

$P(\ominus)$

$P(\oplus | \ominus) = \text{sens.}$

$P(\ominus | \overset{\text{hot}}{\ominus}) = \text{spec.}$

want

$P(\ominus | \oplus)$

$P(\overset{\text{hot}}{\ominus} | \ominus)$

$$P(A|B) \stackrel{?}{=} P(B|A) \quad (10)$$

$P(\text{rain} | \text{clouds overhead}) = \text{low to in between}$

$P(\text{clouds overhead} | \text{rain}) = \text{high}$

\swarrow precision sensitivity

$$P(\ominus | \oplus) = \frac{P(\ominus) P(\oplus | \ominus)}{P(\oplus)} \quad (11)$$

(annoying) normalizing constant

method II / $P(\text{not } \ominus | \oplus) = \frac{P(\text{not } \ominus)P(\oplus | \text{not } \ominus)}{P(\oplus)}$ (8)

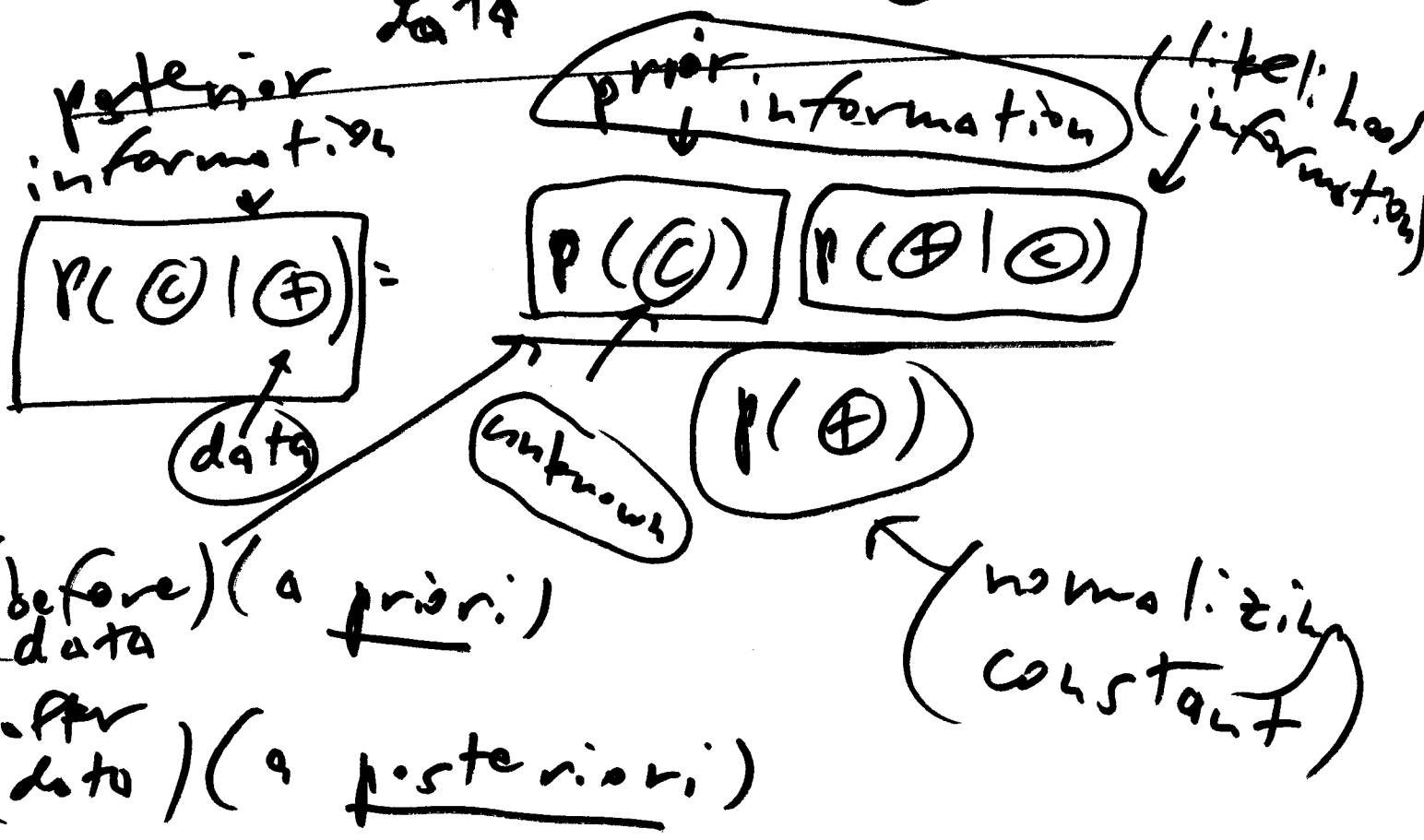
(2)

(1) $\frac{P(\ominus | \oplus)}{P(\text{not } \ominus | \oplus)} = \frac{P(\ominus)}{P(\text{not } \ominus)} \left[\frac{P(\oplus | \ominus)}{P(\oplus | \text{not } \ominus)} \right]$

(2) $\frac{P(\ominus | \oplus)}{P(\text{not } \ominus | \oplus)} = \frac{P(\ominus)}{P(\text{not } \ominus)} \left[\frac{P(\oplus | \ominus)}{P(\oplus | \text{not } \ominus)} \right]$

(Likelihood ratio)

(posterior odds w/ N in favor of \ominus given data) = (prior odds ratio in favor of \ominus) (Bayes factor)



$$p(\text{posterior info}) \propto \frac{p(\text{prior info}) \cdot p(\text{likelihood info})}{\text{normalizing constant}}$$

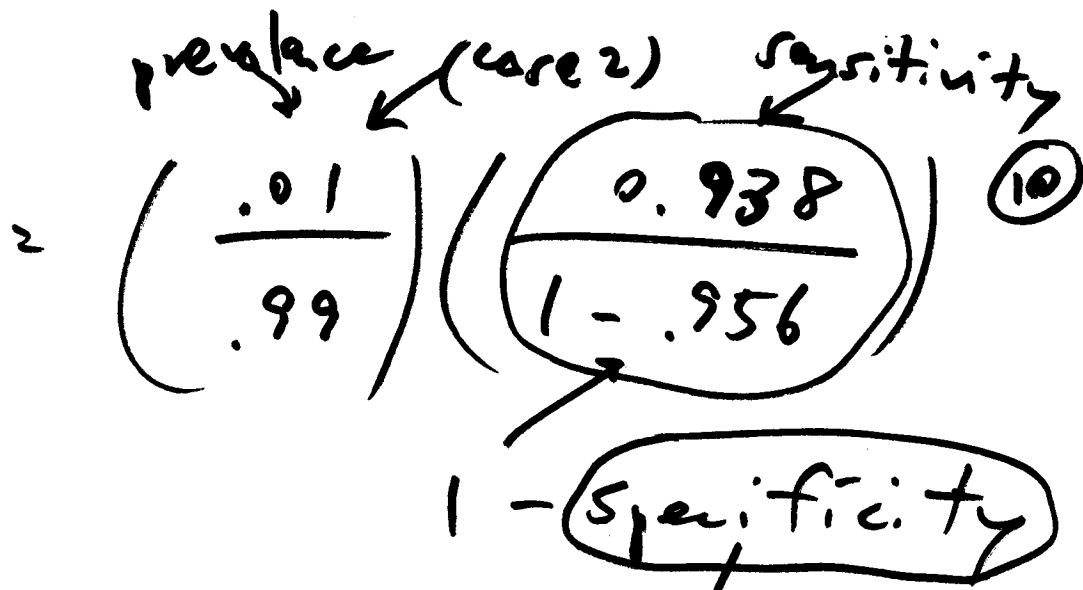
(normalizing constant)

$$p(\text{unknown} | \text{data}) =$$

$$\frac{p(\text{unknown}) \cdot p(\text{data} | \text{unknown})}{\text{normalizing constant}}$$

(normalizing constant)

$$\frac{P(\ominus | \oplus)}{P(\ominus^{\text{not}} | \oplus)}$$



$$P(\oplus | \ominus^{\text{not}}) = 1 - P(\ominus | \ominus^{\text{not}})$$

$$\left(\frac{1}{99} \right) \left(\frac{469}{22} \right)$$