for any event (true/false proposition), let \( P(A) = p_A \), then the odds ratio in favor of \( A \) being true is defined to be \( o_A = \frac{P(A)}{P(\neg A)} = \frac{p_A}{1 - p_A} \)

You can readily solve backwards to get \( p_A = \frac{o_A}{1 + o_A} \) = probability in terms of odds ratio

<table>
<thead>
<tr>
<th>( p_A )</th>
<th>( o_A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>( \frac{1}{99} )</td>
</tr>
<tr>
<td>0.1</td>
<td>( \frac{1}{9} )</td>
</tr>
<tr>
<td>0.5</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>0.9</td>
<td>( \frac{9}{1} )</td>
</tr>
<tr>
<td>0.99</td>
<td>( \frac{99}{1} )</td>
</tr>
</tbody>
</table>

Odds ratios are used by people gambling on things like horse races, but they're also useful in medical research.
\[ \begin{align*}
\text{1. } & \quad p(C \mid E) = \frac{p(C \text{ and } E)}{p(E)} \\
\text{2. } & \quad p(E \mid C) = \frac{p(E \text{ and } C)}{p(C)} \\
\text{3. } & \quad p(E) > 0, \quad p(C) > 0
\end{align*} \]
Bayes's Theorem for single events (T/F propositions)

Case Study: Screening for COVID-19

Truth: 
- COVID-19 positive
- COVID-19 negative

What does test say?
- Test says +
- Test says -

What does test really say?
- COVID-19 positive
- COVID-19 negative

P(CIE) = \frac{P(C) P(E|C)}{P(E)}
have to know 3 things: $P(\text{+}) = 0.177$, $P(\text{+|C}) = 0.956$, $P(\text{+|not C}) = 0.096$.

\[
\begin{align*}
\text{(3)} & \quad P(\text{+|1+}) = 0.956 \\
\text{(O)} & \quad P(\text{+|0+}) = 0.096 \\
\text{(O)} & \quad P(\text{+|0-}) = 0.096 \\
\text{Case 1} & \quad P(\text{+}) = 0.177 \\
\text{Case 2} & \quad P(\text{+}) = 0.177
\end{align*}
\]

Sensitivity = $P(\text{+|C}) = 0.956$, Specificity = $P(\text{+|not C}) = 0.096$.

Prevalence of COVID-19 is $\frac{644116}{32870000} = 0.020$.

$P(\text{+}) = 0.177$, $P(\text{+|C}) = 0.956$, $P(\text{+|not C}) = 0.096$.
Cell line concentration

Case 2

\[ P(C) = 0.002 \]

\[ P(C | +) = \frac{45,768}{1,876 + 45,768} = 0.04 \]

\[ P(\text{not } C | -) = \frac{954,088}{954,088 + 954,212} = 0.99987 \]

| Case | \( P(C | +) \) | \( P(\text{not } C | -) \) | prevalence |
|------|----------------|--------------------------|------------|
| 1    | 0.04%          | 99.99%                   | 0.2%       |
| 2    | 1.18%          | 99.93%                   | 1.0%       |

Cell line test almost perfect when \( \Theta \)
but it's day when it says  

<table>
<thead>
<tr>
<th>True</th>
<th>False</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive</td>
<td>Positive</td>
</tr>
<tr>
<td>False</td>
<td>True Negative</td>
</tr>
</tbody>
</table>

person told  
but really  

\[ A = \{ a_1, a_2, a_3 \} \]

give quicker, cheaper to everybody & pretend it's right

celllex: if  

conclude if \( \Theta \), go to standard & believe it's right

celllex: if  

slow, more costly
Know:

\[ P(\Theta) \]
\[ P(\Theta | \Theta) = \text{seep.} \]
\[ P(\Theta | \text{not } \Theta) = \text{seep.} \]

\[ P(A | B) = P(B | A) \]

\[ P(\text{rain} | \text{clouds}) = \text{low to in between} \]
\[ P(\text{clouds} | \text{rain}) = \text{high} \]

\[ P(\Theta | \Theta) = \frac{P(\Theta) P(\Theta | \Theta)}{P(\Theta)} \] (1)
\[ p(\Theta |+\Theta) = \frac{p(\Theta)p(+|\Theta)}{p(+)} \] (2)

\[ \frac{1}{2} = \frac{p(\Theta |+\Theta)}{p(\Theta |\text{not}+\Theta)} = \frac{\left[p(\Theta)\right]}{\left[p(\Theta |\text{not}+\Theta)\right]} \left[\frac{p(+|\Theta)}{p(+ |\text{not}+\Theta)}\right] \]

(Posterior odds with \text{likelihood ratio})

(Posterior with \text{non} data)

(Prior \text{info} of \Theta)

(likelihood info)

\[ p(\Theta |+\Theta) = \frac{p(\Theta) p(+ |\Theta)}{p(+)} \] (a posteriori)

\[ p(\Theta |\text{not}+\Theta) = \frac{p(\Theta) p(\text{not}+ |\Theta)}{p(\text{not}+)} \] (before data or \text{a priori})

\[ p(\Theta) = \text{normalize constant} \]
\[
\text{posterior} = \frac{\text{likelihood} \cdot \text{prior}}{\text{normalize constant}}
\]

\[
p(\text{unknown} \mid \text{data}) = \frac{p(\text{unknown}) \cdot p(\text{data} \mid \text{unknown})}{\text{normalize constant}}
\]
\[ \frac{P(\Theta|\Theta^+)}{P(\Theta^+|\Theta)} = \left( \frac{0.01}{0.99} \right) \left( \frac{0.938}{1 - 0.956} \right) \]

\[ P(\Theta|\Theta^+) = 1 - P(\Theta|\Theta^+) \]

\[ \left( \frac{1}{0.99} \right) \left( \frac{46.9}{22} \right) \]