Discussion Section week of 30 Mar - 3 Apr 20

STAT 131
30 Mar 20

A brief & intuitive calculus review

(DD) 1

~~read out loud~~ "y equals f of x"

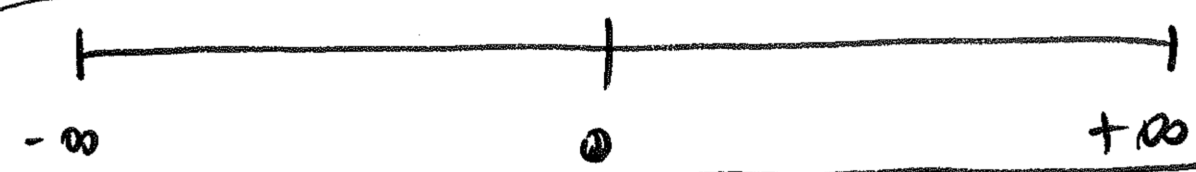
$\boxed{1}$ A function $y = f(x)$ of one (input) variable x is (at an intuitive level) a black-box machine:

you input

$$x \longrightarrow \boxed{f} \longrightarrow y = f(x)$$

and out comes $f(x)$

f accepts a single input x and (crucially) outputs a single output y. In general x and y could be just about anything, but in STAT 131, x and y will (almost) always be real numbers on the extended number line



that includes $-\infty$ on the left and $+\infty$ on the right; in fancy math symbols this set is called $\overline{\mathbb{R}}$ ("$\mathbb{R}$ bar") or $[-\infty, +\infty]$ include endpoints

or $\mathbb{R} \cup \{-\infty, +\infty\}$, and $\mathbb{R} = (-\infty, +\infty)$ don't include endpoints

union

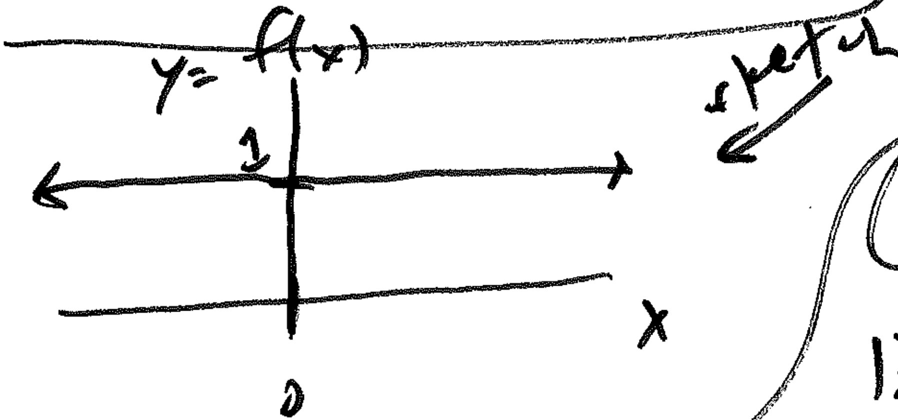
is the usual number line without the infinities.

$\boxed{2}$ whenever possible you should get into the habit of understanding a function in 2 ways: $\boxed{\text{algebraic}}$ (e.g., $f(x) = x^2$) and

geometric (draw a sketch of the function)

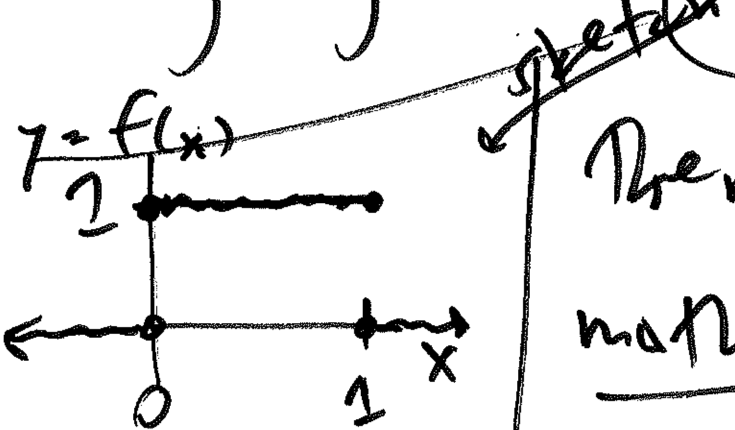
3 simple examples that occur in STAT 131:

(A) $f(x) = 1$



(B) lots of STAT 131 functions are defined in pieces, using computer-science, if-then-else language:

$$f(x) = \begin{cases} 1 & \overset{(\text{if})}{\text{for}}\ 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases} \qquad (*)$$



There is an equivalent math way to say the same thing:

People have found it (highly) useful to create something called an <u>indicator function</u>, to indicate membership in a particular <u>set</u>:

$$I_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & x \notin A \end{cases}$$

$x \in A$ "x is in A" means x is an element of the set A

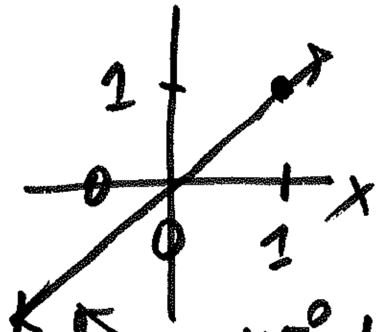
with this new notation the computer-science function ⊛ (p. 3) can equally well be written

$$\boxed{f(x) = 1 \cdot I_{[0,1]}(x)}$$

we will move back & forth interchangeably between these two ways to denote a function in STAT 131.

Examples (continued)

C $f(x) = x$ (linear) sketch

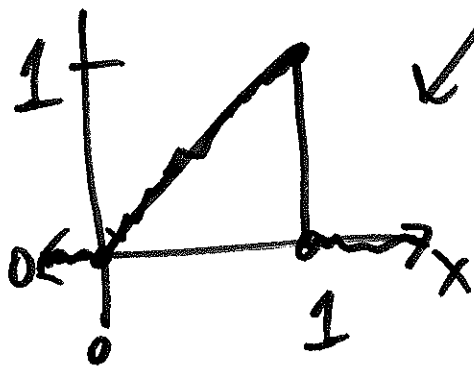


and $f(x) = \begin{cases} x & \text{for } 0 \le x \le 1 \\ 0 & \text{else} \end{cases}$

else: another word for "otherwise"

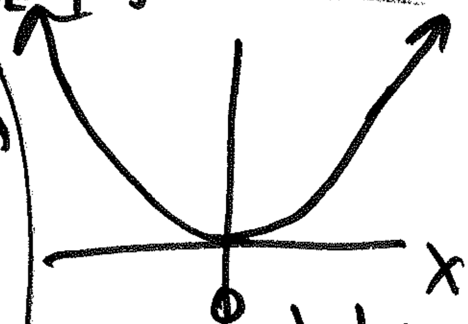
$= x \cdot I_{[0,1]}(x)$

sketch



D sketch

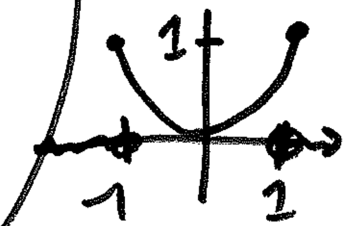
$f(x) = x^2$ (quadratic)



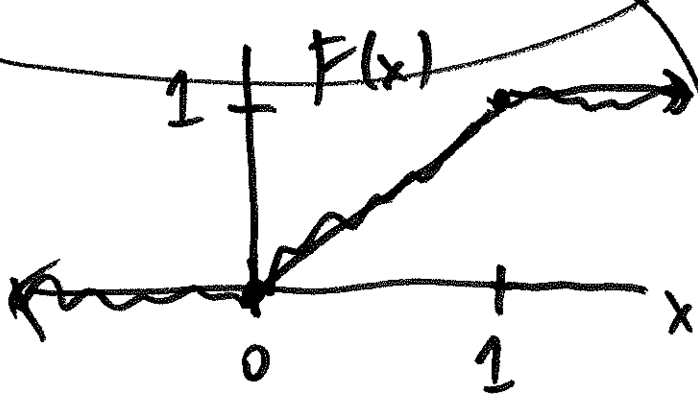
and $f(x) = \begin{cases} x^2 & \text{for } -1 \le x \le +1 \\ 0 & \text{else} \end{cases}$

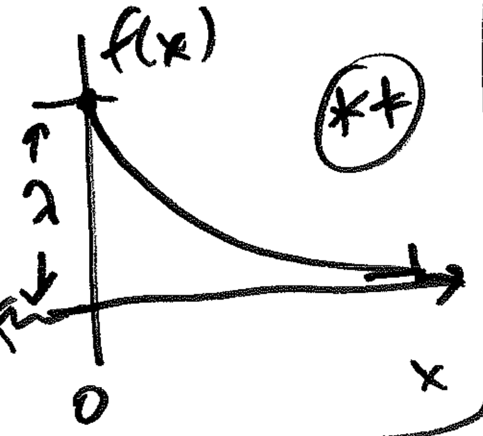
$= x^2 I_{[-1,+1]}(x)$

sketch



E $F(x) = \begin{cases} 0 & \text{for } x \le 0 \\ x & 0 \le x \le 1 \\ 1 & x \ge 1 \end{cases}$





(F) Often in STAT 131 we work with what are called <u>parametric families</u> of functions, <u>indexed by</u> one or more <u>parameters</u> (a <u>parameter</u> is just a real number that specifies which <u>member</u> of the parametric <u>family</u> you're thinking about).

Example: (sketch (**) above)

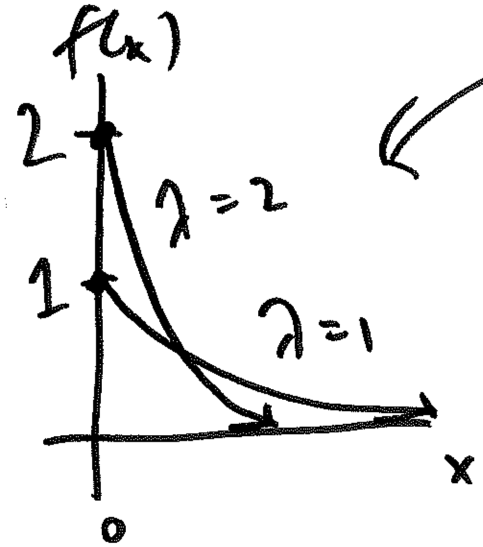
for $\lambda > 0$

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x \geq 0 \\ 0 & \text{else} \end{cases}$$

(***)

Sometimes we write this as $f(x \mid \lambda)$ to emphasize the dependence on λ

"given"



Here are two members of the parametric family (***) (p. 6) plotted on the same graph.

$\boxed{4}$ If you find it hard to visualize a function $y = f(x)$ (this is called a <u>real-valued function of a real variable</u>), get $\boxed{\text{wolframalpha}}$ to plot it for you.

real number

abbreviation Wα in this class

$\boxed{\text{W}\alpha \text{ demo.}}$

At first you may find it hard to communicate with Wα, to get it to do what you want.

when you get stuck, ask google for help. Example Computer-science if-then functions are called piecewise functions.

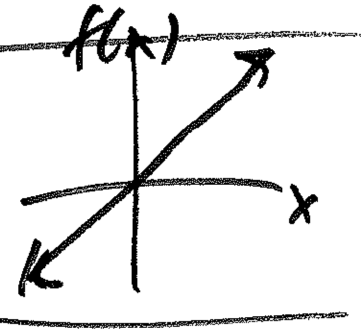
So I used the search string wolfram alpha how to define a piecewise function (all 1 line at google) and I got immediate help. $\boxed{5}$ Some functions are nice, others not so much.

There are a variety of ways to define nice. $\boxed{5A}$ The most basic is continuity: intuitively, $f(x) = y$ is continuous at an input point x_0 if, as x gets closer & closer to x_0, $f(x) = y$ gets closer & closer to $f(x_0)$. This is formalized with the idea of <u>limits</u>:

$f(x)$ is <u>continuous</u> at $x = x_0$

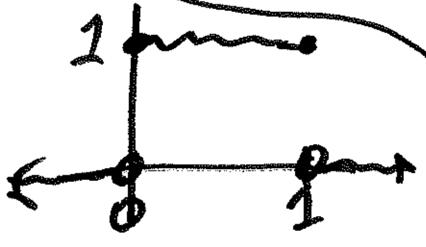
iff (if and only if) $\boxed{\lim_{x \to x_0} f(x) = f(x_0)}$

Example: $\boxed{f(x) = x \text{ for all } x \in (-\infty, \infty)}$



is continuous for all $-\infty < x < +\infty$,

but $f(x) = \begin{cases} 1 & \text{for } 0 \le x \le 1 \\ 0 & \text{else} \end{cases}$ is discontinuous at $x = 0$ and $x = 1$



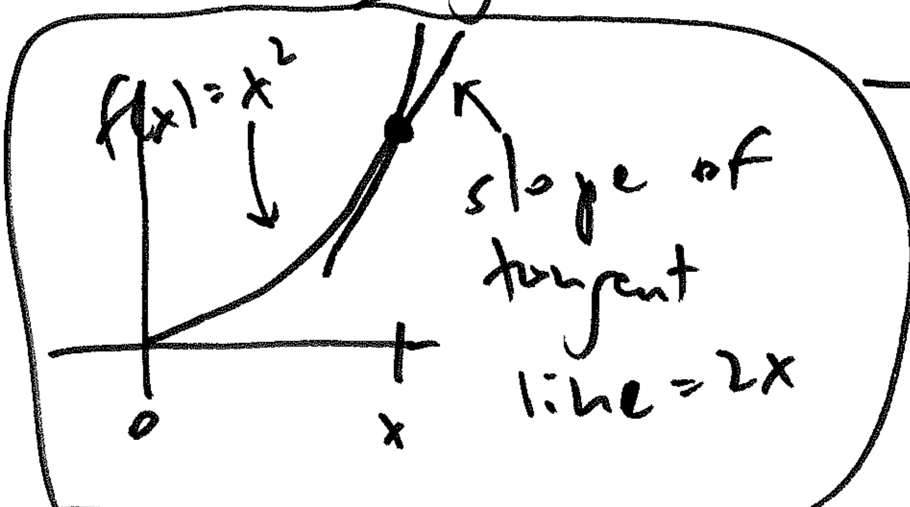
$\boxed{5B}$ Another definition of $\boxed{\text{nice}}$ is

<u>differentiability</u>: does $f(x)$ have a <u>derivative</u> at $x = x_0$? You will recall that

$$f'(x_0) = \left[\frac{d}{dx} f(x)\right]_{x = x_0}$$

"at"

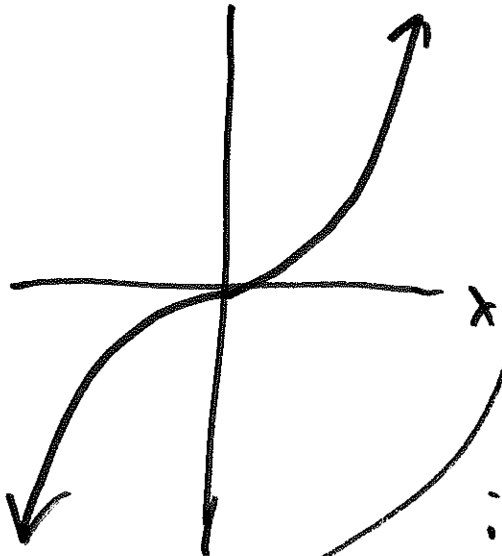
is the <u>slope</u> of the <u>tangent line</u> to $f(x)$ at $x = x_0$



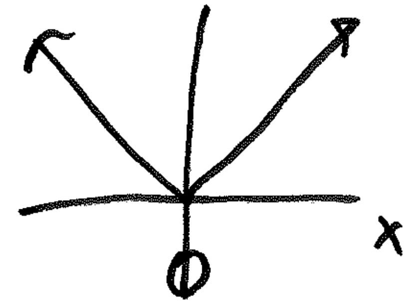
Informally, $f(x)$ is differentiable at $x = x_0$ iff

$$\left(\text{The slope of the tangent line when approaching } x_0 \text{ from below (the left)}\right) = \left(\text{the slope of the tangent line when approaching } x_0 \text{ from above (the right)}\right)$$

Examples $f(x) = x^3$ is differentiable for all $x \in (-\infty, +\infty)$



but $f(x) = |x|$ (absolute value of x) is only differentiable for all $x \neq 0$ (at $x = 0$, left slope -1, right slope $+1$)



Advice You will need to compute the derivatives of a (wide) variety of functions in STAT 131; it would be good for you to review (e.g., online) the basic rules of differentiation this week.

Examples | 1 $f(x) = x^t$ for any real number t

$\to \frac{d}{dx} f(x) = f'(x) = t\,x^{t-1}$

0 $f(x) = c$

$\to f'(x) = 0$

for all real c

2 $f(x) = c \cdot g(x) \to$

$f'(x) = c \cdot g'(x)$

3 $f(x) = g(x) \pm h(x)$

$\to f'(x) = g'(x) \pm h'(x)$

4 $f(x) = c\,e^{tx} \to f'(x) = c\,t\,e^{tx}$

5 $f(x) = \ln x \to f'(x) = \frac{1}{x}$

Advice | when computing a derivative is hard, give it to $\boxed{\text{Wa}}$

(30 Mar/20)

$\boxed{\text{Wa}}$ command:

Example | differentiate c * exp(t * x) in x