STAT 131
20 May 20

catch-up lecture

this time: variance, SD,

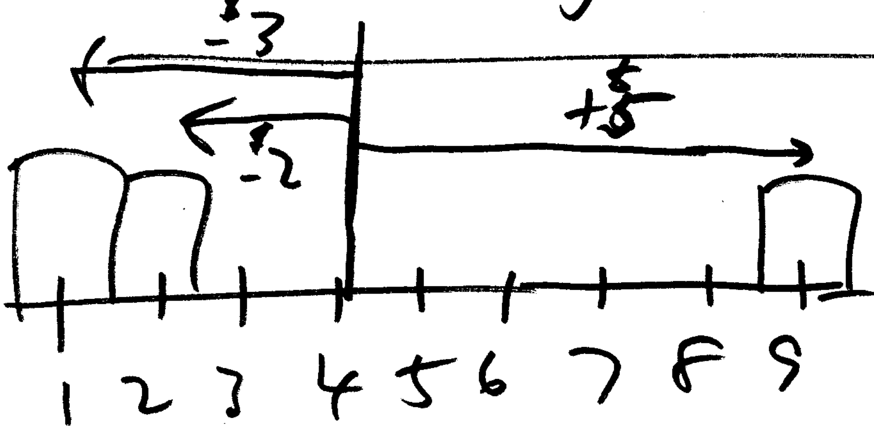
next time: covariance, correlation

THT 2 now due next Wed 27 May 20

Quiz 7 now due Sun 24 May 20

(1)

I will now start my daily office 1.5-hour sessions, from tomorrow through next Wed 27 May 20



$$\underline{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \qquad n = 3$$

(sample) mean $\bar{y} = 4$ → data

$E(\bar{Y})$ ← random variable

← expected value

$$\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \xrightarrow{\text{subtract}} 4 \begin{bmatrix} -3 \\ -2 \\ +5 \end{bmatrix}$$

mean 0

$$\begin{bmatrix} Y_1 \\ \vdots \\ Y_n \end{bmatrix} \xrightarrow{\text{subtract}} \bar{y} \begin{bmatrix} Y_1 - \bar{y} \\ \vdots \\ Y_n - \bar{y} \end{bmatrix}$$ ← deviations from mean

first thought

mean

$$\begin{bmatrix} y_1 - \bar{y} \\ \vdots \\ y_n - \bar{y} \end{bmatrix} \rightarrow$$

mean $\underline{\underline{0}}$

2^{nd} thought

$$\begin{bmatrix} |y_1 - \bar{y}| \\ \vdots \\ |y_n - \bar{y}| \end{bmatrix} \begin{matrix} |-3| \\ |-2| \\ |+5| \end{matrix}$$

MAD $\frac{10}{3} = 3.3$

mean MAD (AAD)

not used much today

mean absolute deviation from mean

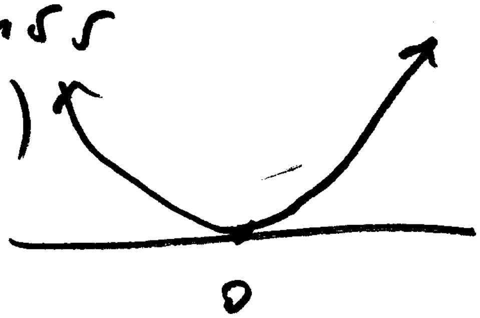
Laplace (1775)

or

$$\begin{bmatrix} (y_1 - \bar{y})^2 \\ \vdots \\ (y_n - \bar{y})^2 \end{bmatrix}$$

mean $\frac{1}{n} \sum_{i=1}^{n} (y_i - \bar{y})^2$

Gauss (1800)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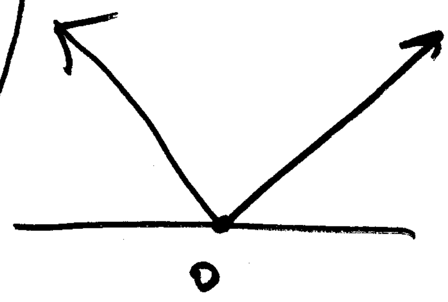
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not diff. at 0

$$\sum_{i=1}^{n} (y_i - \bar{y})^2 \frac{1}{n}$$

weights

$$\begin{bmatrix} 1 \\ 2 \\ 2 \\ 9 \end{bmatrix}$$

y	$P(Y=y)$
1	1/4
2	1/2
9	1/4

$$\sum_{\text{all } y \in S_Y} [y - E(Y)]^2 P(Y=y) = V(Y)$$

variance

③

Y discrete $\rightarrow$ $V(Y) = \sum_{\substack{\text{all} \\ y \in S_Y}} [y - E(Y)]^2 \cdot p(Y = y)$

Y continuous $\rightarrow$ $\boxed{V(Y) = E\,(Y - \mu_Y)^2}$

$= \int_{\substack{\text{all } y \\ \in S_Y}} [y - E(Y)]^2 f_Y(y)\,dy$

$\$$

$\mu_Y = E(Y)$

Gauss's variance idea:

good (math easier; variance plays well with bell curve)

bad wrong units

to get $V(Y)$ back on right scale,

$SD(Y) \doteq \sqrt{V(Y)}$

$\leftarrow$ standard deviation

$$V(X) = E\left[(X - \underbrace{\mu_X}_{E(X)})^2\right]$$

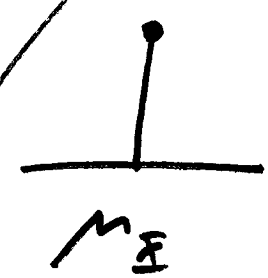
$$= E\left[X^2 - 2X\mu_X + \mu_X^2\right]$$

$$= E(X^2) + E\big((-2)\,X\,\mu_X\big) + E(\mu_X^2)$$

random

constants

$$= E(X^2) + (-2\mu_X)\underbrace{E(X)}_{\mu_X} + \mu_X^2$$



$$= E(X^2) - 2\mu_X^2 + \mu_X^2$$

$$= E(X^2) - \mu_X^2$$

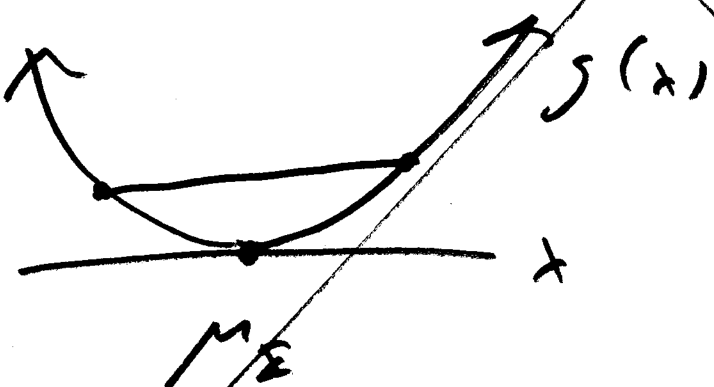
$$V(X) = \underbrace{E(X^2) - \left[E(X)\right]^2}$$

compute this with this

$$V(X) = E\left[(X-\mu_X)^2\right]$$

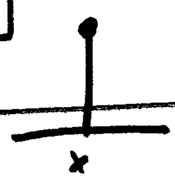
$$= E[g(X)],$$

$$g(X) = (X-\mu_X)^2, \quad g(x) = (x-\overset{3}{\mu_X})^2$$



bowl-shaped up = convex

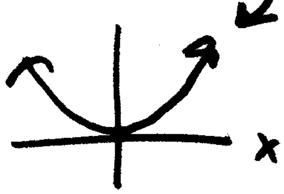
Jensen: $E[g(X)] \overset{?}{\geq} g[E(X)]$



$\boxed{V(X)=0}$

$$V(X) = E(X^2) - [E(X)]^2 \geq 0 \; ✓$$

$g(x) = x^2$



so $E[g(X)] \geq g[E(X)]$

$$E(X^2) \geq (E(X))^2$$

Rule 0

$$V(c) = 0$$

$$SD(c) = 0$$

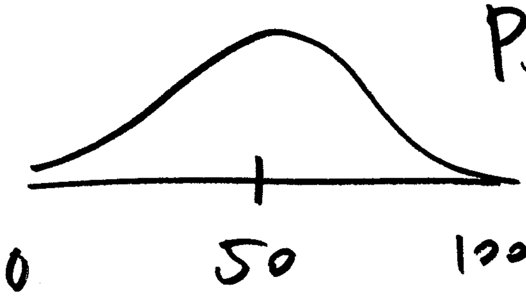
constant

Rule 1

$$V(X + c) = V(X)$$

$$SD(X + c) = SD(X)$$

$$Y = X + 10$$



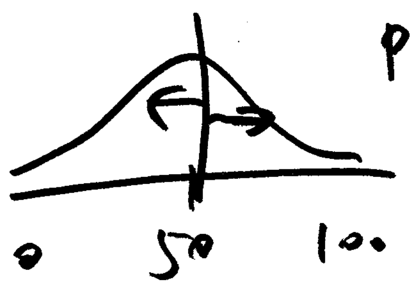
PDF of X $\rightarrow$

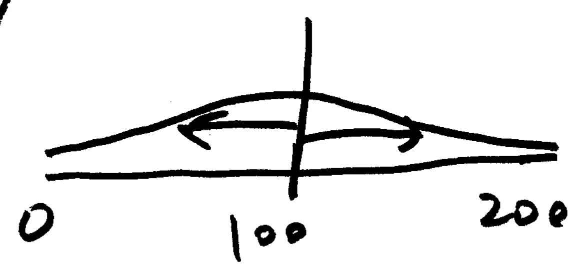
Rule 2

$$SD(cX) = |c|\,SD(X)$$

$$Y = \underline{\underline{2}}X$$

PDF of X





SD $\underline{\underline{18}}$

$$V(cX) = c^2 V(X)$$

$$V(aX + b) = V(aX) = a^2 V(X)$$

$$E\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} E(X_i)$$ for

all X_i, independent or not

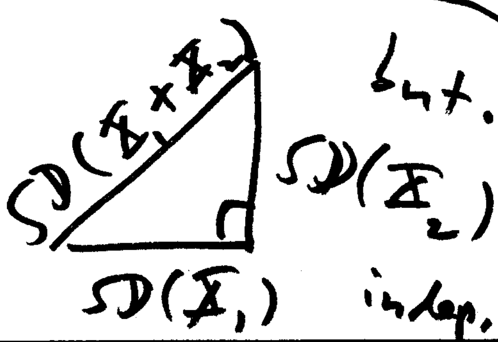
$$V\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} V(X_i)$$ but

only if the X_i are independent

X_1, X_2 indep. $\rightarrow$ $V(X_1 + X_2)$

$$= V(X_1) + V(X_2)$$

$$SD^2(X_1 + X_2) = [SD(X_1)]^2 + [SD(X_2)]^2$$



but only if X_1, X_2 indep.

$$\max\begin{pmatrix} SD(X_1), \\ SD(X_2) \end{pmatrix} \le SD(X_1 + X_2) \le SD(X_1) + SD(X_2)$$

when X_1, X_2 indep

$$\begin{pmatrix} \text{uncertainty} \\ \text{in predicting } X \end{pmatrix} \longleftrightarrow SD(X)$$