DD office 1.5 hour

quiz 2 problem 2

STAT 131
9 Apr 20

$P(1^{st} \text{ ball } R) = \frac{r}{n}$ (red)

ELM? yes

"at random"

① marginal behavior of data generating process

$P(\frac{n}{2} \text{ ball } R) = \frac{r}{n}$ in slot $\frac{n}{2}$

ex. $n = 14$ $7 = \frac{n}{2}$

R: 1, 3, ..., 13

not R: 2, 4, ..., 14

47 3 — — — — — — — — ... — — —

1 2 3 $\frac{n}{2}$ n

$\mathbb{P}_1$

process hint: if the current problem is hard, try to find another problem $\mathbb{P}_2$ that is (a) easy & (b) identical in structure to $\mathbb{P}_1$

$P(\text{last ball } R) = \frac{r}{n}$

$P(2^{nd} \text{ ball } R \mid 1^{st} \text{ ball } R) = \frac{r-1}{n-1}$

$P(2^{nd} \text{ ball } R) = \frac{r}{n}$

(2)

$$P(B \mid A) = \begin{cases} \dfrac{P(A \text{ and } B)}{P(A)} & \text{if } P(A) > 0 \\ \text{undefined} & P(A) = 0 \end{cases}$$

filling slots

(1) $\underline{7} \cdot \underline{2} = 14$ ✓

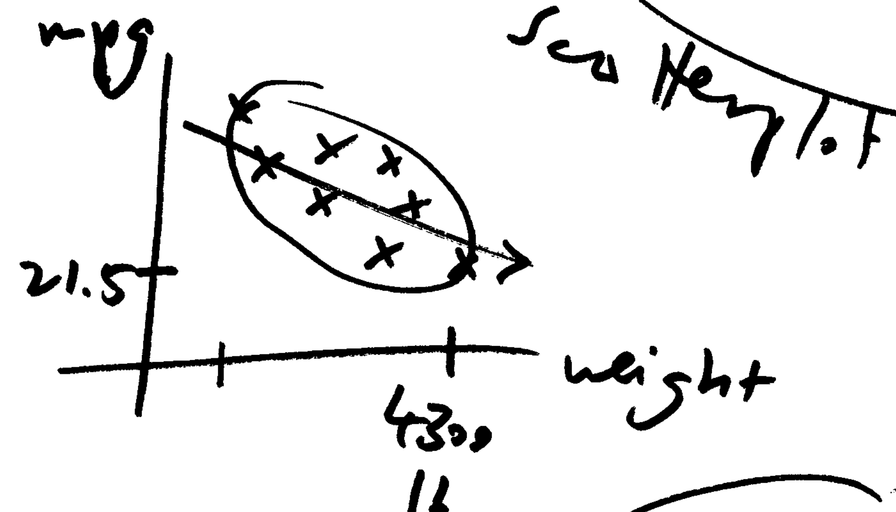
day of week

leap year or not

1(a) - 0
1(b)(i) - 3
1(b)(ii) - 0
⋮

-21

total: 330
-21
309



make	model	weight	mpg
BMW	5	4300 lb	21.5 mpg

weight ↑ mpg ↓ negative association

$P(A) = p_A \leftrightarrow$ odds ratio o_A in favor of A being true: $o_A = \dfrac{p_A}{1-p_A}$

$= \dfrac{P(A)}{P(\text{not}A)}$

ex.

p_A	o_A
0.1	$\frac{0.1}{0.9} = \frac{1}{9}$ odds in favor of A = 9 to 1 odds against A
0.5	1 (even money proposition) $= \frac{0.5}{0.5}$ (50%/50%)
0.9	$\frac{0.9}{0.1} = 9$ to 1 odds in favor of A

equivalent def.

odds ratio $\bar{o}_A$ against A being true: $\bar{o}_A = \dfrac{1-p_A}{p_A} = \dfrac{P(\text{not}A)}{P(A)}$

("powerball "odds") are actually probabilities, not odds

$$p\left(\begin{matrix}\text{Grand}\\ \text{prize}\end{matrix}\right) = \frac{1}{292201338}$$

$p($all 5 w right and B right$)$

$p(A \text{ and } B) = p(A) \cdot p(B)$

but only if A, B independent

$p(A \text{ or } B) = p(A) + p(A)$,

but only if A, B are mutually exclusive

5

$$P\left(\begin{matrix}\text{Grand}\\ \text{prize}\end{matrix}\right) = P\left(\begin{matrix}\text{all 5}\\ W\end{matrix}\text{ right and }\begin{matrix}\text{all 1}\\ R\end{matrix}\text{ right}\right)$$

$$\overset{I}{=} P\left(\begin{matrix}\text{all 5}\\ W\end{matrix}\text{ right}\right) \cdot P(R \text{ right})$$

$$P\left(\begin{matrix}\text{all 5}\\ W\end{matrix}\text{ right}\right)$$

W bowl: 69 numbers

R : 26

population = { all balls in W bowl }

$N = 69$

1
2
3
4
5

$n_1 = 5$

5 balls drawn by PowerBall

6
⋮
69

64 balls not drawn

$N_1 = 5$

$N_2 = 64$

$n_2 = 0$

$$P\left(\begin{matrix}\text{all 5}\\ W\end{matrix}\text{ right}\right) = \frac{\binom{N_1}{n_1}\binom{N_2}{n_2}}{\binom{N}{n}} = \frac{\binom{5}{5}\binom{64}{0}}{\binom{69}{5}}$$

⑥

$$= \frac{\left(\frac{5!}{0!\,5!}\right)\left(\frac{64!}{0!\,64!}\right)}{\frac{69!}{64!\,5!}} = \frac{64!\,5!}{69!}$$

$$= \frac{5\cdot4\cdot3\cdot2\cdot1\ \cancel{64!}\cdot\cancel{5!}}{69\cdot68\cdot67\cdot66\cdot65\cdot\cancel{64!}}$$

$$P(\text{R right}) = \frac{1}{11238513}$$

$$= \frac{1}{26} = \frac{\binom{1}{1}\binom{25}{0}}{\binom{26}{1}}$$

$$P\left(\begin{matrix}\text{Grand}\\\text{prize}\end{matrix}\right) = \frac{1}{11238513\cdot26} = \frac{1}{292201338}$$