quiz 9

$X_1, \dots, X_n \overset{IID}{\sim} \begin{cases} E(X_i) = \mu \\ V(X_i) = \sigma^2 < \infty \end{cases}$

STAT 131
7 Jun 20

?

$\bar{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i$

Too little data possible?

DD extra office 1.5-hour session

yes

too much data possible?

①

~~estimate of μ~~

Chebyshev

$$\underline{P(|\bar{X}_n - \mu| \le k\sigma) \ge} \text{ close to } 1 \quad (**)$$

want small with high probability

Y : $E(Y)$, $V(Y)$ exists

pick any $t \ge 0$

$$P(|Y - E(Y)| \ge t) \le \frac{V(Y)}{t^2}$$

($k > 0$, $\sigma > 0$)

$E(\bar{X}_n) = \mu$

$(*)$

$$\underline{P(|\bar{X}_n - \mu| \ge k\sigma)} \le \frac{\sigma^2/n}{k^2\sigma^2}$$

$$= \frac{1}{nk^2}$$

$V(\bar{X}_n) = \frac{\sigma^2}{n}$

$$P(|\bar{X}_n - \mu| > k\sigma)$$

$$\shortparallel$$

$$1 - P(|\bar{X}_n - \mu| \le k\sigma) \le \frac{1}{nk^2}$$

$$P(|\bar{X}_n - \mu| \le k\sigma) \ge 1 - \frac{1}{nk^2}$$

$$P(|\bar{X}_n - \mu| \le k\sigma) \ge \boxed{1 - \frac{1}{nk^2} \ge 1 - \alpha}$$

$0 < \alpha < 1$

small

$$-\frac{1}{nk^2} \ge -\alpha$$

$$\frac{1}{nk^2} \le \alpha$$

$$\boxed{\frac{1}{\alpha k^2} \le n_{chebyshev}}$$

$\alpha \downarrow 0$

$n_{chebyshev} \uparrow$ ✓

$k \downarrow \quad n \uparrow$

$X_i \overset{IID}{\sim} N(\mu, \sigma^2)$

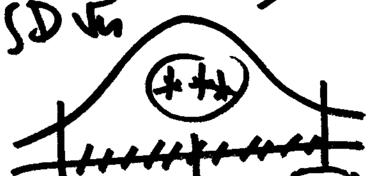
$(i = 1, \dots, n)$

$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i$$

SD $\frac{\sigma}{\sqrt{n}}$

PDF of $\bar{X}_n$

$P(|\bar{X}_n - \mu| < k\sigma)$



$\mu - k\sigma \qquad \mu$

raw units (x)

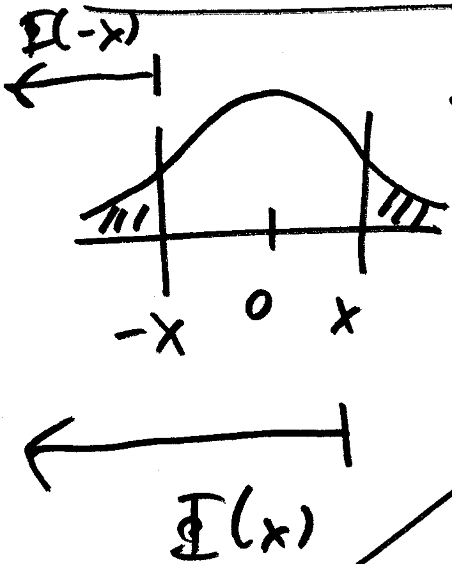
$\mu + k\sigma$

standard units (z)

$-k\sqrt{n} \qquad 0 \qquad k\sqrt{n}$

$$z = \frac{x - \mu}{\sigma/\sqrt{n}} = \frac{(\mu + k\sigma) - \mu}{\sigma/\sqrt{n}} = \frac{k\sigma}{\sigma/\sqrt{n}} = k\sqrt{n}$$

$$(***) = \Phi(k\sqrt{n}) - \Phi(-k\sqrt{n})$$



standard normal pdf

$$\Phi(x) + \Phi(-x) = 1$$

$$(***) = \Phi(k\sqrt{n}) - (1 - \Phi(k\sqrt{n}))$$

$$= 2\Phi(k\sqrt{n}) - 1 \checkmark$$

$\therefore$

$$P(|\bar{X}_n - \mu| < k\sigma) \approx 2\Phi(k\sqrt{n}) - 1 \checkmark$$

$$\geq$$

$$2\Phi(k\sqrt{n}) - 1 = 1 - \alpha$$

$$2\Phi(k\sqrt{n}) = 2 - \alpha$$

$$\cancel{\Phi^{-1}[\Phi}(k\sqrt{n})] = \Phi^{-1}\left(1 - \frac{\alpha}{2}\right)$$

$$n_{chebyshev} \geq \frac{1/\alpha}{k^2}$$

$$n_{normality} \geq \left(\frac{\Phi^{-1}(1 - \frac{\alpha}{2})}{k}\right)^2 = \frac{\left(\Phi^{-1}(1 - \frac{\alpha}{2})\right)^2}{k^2}$$

α	$\frac{1}{\alpha}$	$\left(\Phi^{-1}(1-\frac{\alpha}{2})\right)^2$
0.1	10	2.71
0.05	20	3.84
0.01	100	6.63
0.005	200	7.88
0.001	1000	10.83

@ take a ~~(small)~~ pilot sample of size n_{pilot}; check for approximate normality

$$P(|\bar{X}_n - \mu| < k\sigma) \geq 1-\alpha$$