$$f_{X,Y}(x,y) = \begin{cases} c x^2 & \text{for } 0 \le y \le 1-x^2 \\ 0 & \text{else} \end{cases}$$

STAT 131
5 May 20
DD office 1.5 hr.

quiz 5

to be a bivariate PDF,

1. $\iint_{S_{X,Y}} f_{X,Y}(x,y)\,dy\,dx = 1$
2. $f_{X,Y}(x,y) \ge 0$ for all x,y

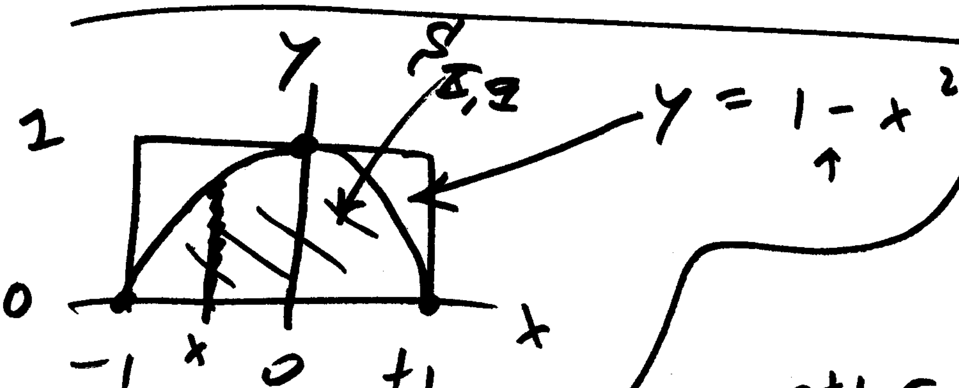
$S_X = ?$ $= [-1,+1]$

$0 \le y \le 1-x^2 \to 0 \le 1-x^2 \to x^2 \le 1 \to -1 \le x \le +1$

$S_Y = ?$ $= [0,1]$

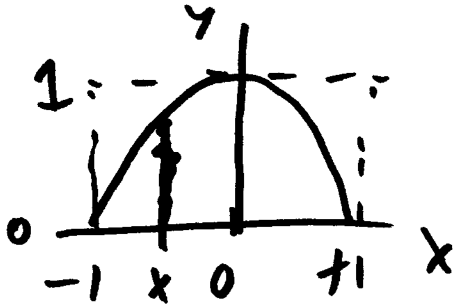
$0 \le y \le \underline{\underline{1-x^2}}$ as x goes from -1 to $+1$,

$y = 1-x^2$ goes from 0 to 1 $\to S_Y = [0,1]$



$$\iint_{S_{X,Y}} c x^2\,dy\,dx = 1$$

$$= \int_{-1}^{+1}\left[\int_0^{1-x^2} c x^2\,dy\right] dx = 1$$



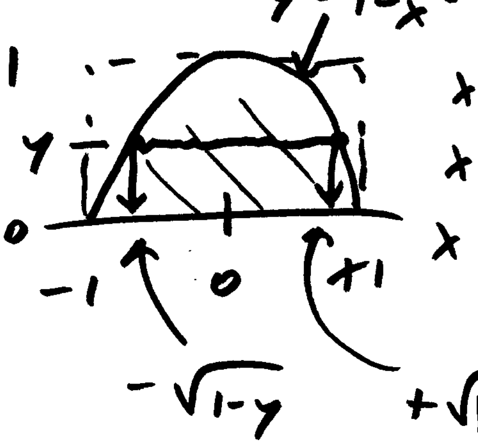
$$f_X(x) = \begin{cases} & \text{for } -1 \le x \le +1 \\ 0 & \text{else} \end{cases}$$

fix an $x \in [-1, +1]$

$$f_X(x) = \int_{S'_{X,Y}} c x^2 \, dy = \int_0^{1-x^2} c x^2 \, dy$$

$f_Y(y) = ?$ fix a $y \in [0, 1)$

$$f_Y(y) = \int_{S'_{X,Y}} c x^2 \, dx = \int_{-\sqrt{1-y}}^{+\sqrt{1-y}} c x^2 \, dx$$

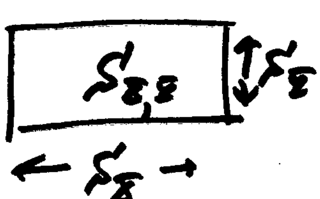


$x^2 = 1 - y$

$x = \pm\sqrt{1-y}$

$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$

set



$S'_{X,Y} = S'_X \times S'_Y$

X, Y independent iff $f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y)$ for all $(x,y) \in \mathbb{R}^2$

if $S_{X,Y} = S_X \times S_Y$

then (X, Y) *may* be indep; see

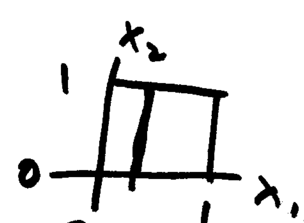
Q: If $S_{X,Y} \neq S_X \times S_Y$, does this prove that X, Y are dependent?

A: I don't know of any counter-examples.

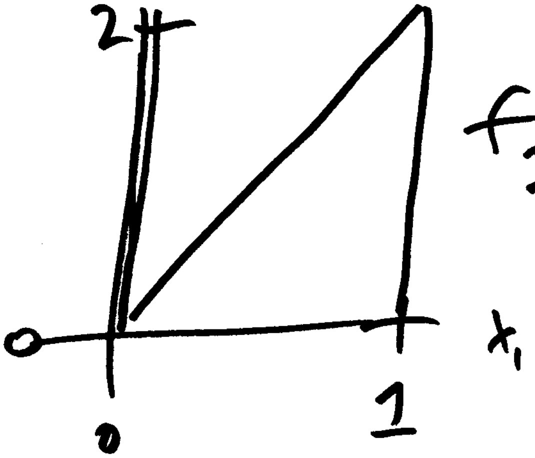
$$f_{X_1,X_2}(x_1, x_2) = \begin{cases} 4x_1x_2 & \text{for} \begin{pmatrix} 0 < x_1 < 1 \\ 0 < x_2 < 1 \end{pmatrix} \\ 0 & \text{else} \end{cases}$$

for fixed $0 < x_1 < 1$

$$f_{X_1}(x_1) = \int_0^1 4x_1x_2 \, dx_2$$



$$= (4x_1)\frac{x_2^2}{2}\Big|_0^1 = 2x_1$$



$f_{X_1}(x_1)$

area = 1 = $\frac{1}{2}(b)(h)$

$= \frac{1}{2}(1)(2) = 1$ ✓

$(x_1, x_2) \rightarrow (y_1, y_2)$

$\uparrow \quad \uparrow$

$3 \quad 7$

$(3, 21)$

$y_1 = x_1$

$y_2 = x_1 \cdot x_2$

prior info. likelihood info.

$$P(u \mid D) = \frac{P(u)\,P(D \mid u)}{P(D)}$$

normalizing constant

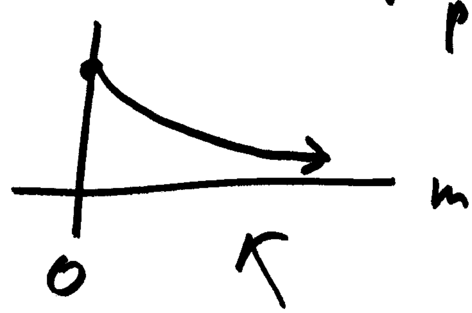
unknown — data

(did you get envelope 1 E_1 or envelope 2?) I

(you see \$x in your envelope)

(actual value of m) ← \$ in E_1

cont. prior distr. PDF



$f_M(m) = p(m)$

you & your friends are poor students

impoverished