126.
package of butter

16 oz
16
16
16

16
16
16
16

16.0
16.0
16.0
16.0

16.01
16.00
15.97

15.98

1.5-hour office session
May 20

probabilistic

stochastic

\theta = \text{true value of the parameter measured}

The observed measurements

\bar{X}_n \sim \text{IID } \mathcal{N}(\theta, \sigma)

\bar{X}_n \sim \text{IID } \mathcal{N}(\theta, \sigma)

\text{error } \epsilon_1 \sim \mathcal{N}(0, \sigma)

\text{error } \epsilon_2 \sim \mathcal{N}(0, \sigma)

\text{obs. } 1 = \text{truth} + \text{bias} + \text{random error}

\text{obs. } 2 = \text{truth} + \text{bias} + \text{random error}

\bar{X}_n = \text{mean of the measurements}

deterministic
\( \mathcal{X}_n = \theta + b + e_n \)

Observables:

\[ \mathcal{Y}_i = \theta + b + e_i \]

\( \mathcal{Y}_i \sim N(\theta + b, \sigma^2) \) for \( i = 1, \ldots, n \)

\( \theta + b + \mathbb{E}(e_i) = \theta + b \)

\( \mathbb{V}(\mathcal{Y}_i) = \mathbb{V}(\theta + b + e_i) \)

\( = \mathbb{V}(e_i) = \sigma^2 \)
\[ Z - X - \theta + b + \epsilon_n \]

\[ \overline{X}_n = \frac{1}{n} \sum_{i=1}^{n} x_i \]

\[ \overline{D}_n = \frac{1}{n} \sum_{i=1}^{n} (\theta + b + \epsilon_i) \]

\[ = \left( \frac{1}{n} \sum_{i=1}^{n} \theta \right) + \left( \frac{1}{n} \sum_{i=1}^{n} b \right) + \left( \frac{1}{n} \sum_{i=1}^{n} \epsilon_i \right) \]

\[ = \theta + b + \epsilon_n \]

\[ \sqrt{V(\overline{D}_n)} = \sqrt{V\left( \frac{1}{n} \sum_{i=1}^{n} \epsilon_i \right)} = \frac{1}{n} \sqrt{V\left( \sum_{i=1}^{n} \epsilon_i \right)} \]

\[ \Theta \equiv \frac{1}{n^2} \sum_{i=1}^{n} \sqrt{V(\epsilon_i)} = \frac{1}{n^2} \sum_{i=1}^{n} \sigma^2 = \frac{n \sigma^2}{n^2} \]

\[ \Rightarrow \sqrt{V(\overline{D}_n)} = \frac{\sigma^2}{\sqrt{n}} \]
$$E(\overline{X}_n) = E\left(\frac{\sum_{i=1}^{n} x_i}{n}\right) = \theta + b + E(\overline{e}_n) = \theta + b$$

$$E(e_i) = E\left(\frac{1}{n}\sum_{i=1}^{n} e_i\right) = \frac{1}{n} E\left(\sum_{i=1}^{n} e_i\right) = \frac{1}{n} \sum_{i=1}^{n} E(e_i) = \frac{1}{n} \sum_{i=1}^{n} 0 = 0$$

$$V(e_i) = \sigma^2$$

$$V(e_i) = \sigma^2$$

$$V(\overline{e}_n) = \frac{\sigma^2}{n}$$

$\overline{X}_n$ has expected value $(\theta + b)$ and variance $\frac{\sigma^2}{n}$.

15.98 = 16.00 + 0 + (-0.02)
16.01 = 16.00 + 0 + (+0.01)
16.00 = 16.00 + 0 + (+0.00)
15.97 = 16.00 + 0 + (-0.03)

\[
\bar{X}_n = \theta + b + \frac{\sigma^2}{n}
\]
\[ Z_i \sim \text{mean } (\theta + 6), \text{ variance } \sigma^2 = \sigma \]
\[ Z_i \sim \text{mean } (\theta + b), \text{ variance } \frac{c^2}{n} \]

\[ \text{WLLN: } \overline{Z}_n \xrightarrow{P} \theta + b \neq \theta \quad \text{unless} \quad b = 0 \]

i.e., unless the measuring process is unbiased,

not automatically guaranteed for all measurement processes.

Different bis every 20 years.

Michaelson & Morley (speed of light in air)

(1890) +

doesn't

\[ C = 300 \text{ m/s} \text{/sec} \]

←big→