"1 lb." package of butter

$$\begin{bmatrix} 16 \text{ oz} \\ 16 \\ 16 \\ \vdots \\ 16 \end{bmatrix} \quad \begin{bmatrix} 16.0 \\ 16.0 \\ 16.0 \\ \vdots \\ 16.0 \end{bmatrix}$$

deterministic

$$\begin{bmatrix} 15.98 \\ 16.01 \\ 16.00 \\ 15.97 \\ \vdots \end{bmatrix}$$

probabilistic or stochastic

STAT 131 1

1) extra office 1.5-hour session (30 May 20) (1)

θ = true value of thing measured

sample

the observed measurements

$$\begin{array}{c} I_1 \\ \vdots \\ I_n \end{array} \begin{bmatrix} Y_1 \\ \vdots \\ Y_n \end{bmatrix} \; n \quad (\bar{Y}_n)$$

mean $\bar{I}_n$ (constants)

error SD = σ

error variance

IID $N(0, \sigma^2)$

"error"

$$I_1 = \theta + b + e_1 \text{ (r.v.)}$$

obs. 1 = truth + bias + random error$_1$

$$I_2 = \theta + b + e_2 \text{ (r.v.)}$$

obs$_2$ = truth + bias + random error$_2$

$$Y_n = \theta + b + e_n$$

$$\text{obs}_n = \text{truth} + \text{bias} + \text{random error}_n$$

← unobservable →

IID $N(0, \sigma^2)$

$$Y_i = \theta + b + e_i$$

$(i = 1, \ldots, n)$

only observable Y_i

$E(e_i) = 0$

$V(e_i) = \sigma^2$

independent

$$\boxed{Y_i \overset{\text{IID}}{\sim} N(\theta + b, \sigma^2)}$$

$(i = 1, \ldots, n)$

$$E(Y_i) = E(\theta + b + e_i)$$

$$= \theta + b + \cancelto{0}{E(e_i)} = \theta + b$$

$$V(Y_i) = V(\underline{\underline{\theta + b}} + e_i)$$

$$= V(e_i) = \sigma^2$$

TMT 3

1 (a)

$$Y_i = \theta + b + e_i$$

$$\bar{Y}_n = \frac{1}{n}\sum_{i=1}^{n} Y_i$$

$$\bar{Y}_n = \frac{1}{n}\sum_{i=1}^{n}(\theta + b + e_i)$$

$$= \left(\frac{1}{n}\sum_{i=1}^{n}\theta\right) + \left(\frac{1}{n}\sum_{i=1}^{n} b\right) + \left(\frac{1}{n}\sum_{i=1}^{n} e_i\right)$$

$$= \frac{n\theta}{n} + \frac{nb}{n} + \frac{1}{n}\sum_{i=1}^{n} e_i$$

$$= \theta + b + \bar{e}_n$$

$$V(\bar{e}_n) = V\left(\frac{1}{n}\sum_{i=1}^{n} \overset{\text{r.v.}}{e_i}\right) = \frac{1}{n^2} V\left(\sum_{i=1}^{n} e_i\right)$$

$$\overset{\text{IID}}{=} \frac{1}{n^2}\sum_{i=1}^{n} V(e_i) = \frac{1}{n^2}\sum_{i=1}^{n}\sigma^2 = \frac{n\sigma^2}{n^2}$$

so $V(\bar{e}_n) = \dfrac{\sigma^2}{n}$

$$E(\bar{Y}_n) = E(\underbrace{\theta + b}_{\text{constants}} + \overset{\text{r.v.}}{\bar{e}_n})$$

$$= \theta + b + E(\bar{e}_n) = \theta + b$$

$$E(\bar{e}_n) = E\left(\frac{1}{n}\sum_{i=1}^{n} e_i\right)$$

$$= \frac{1}{n} E\left(\sum_{i=1}^{n} e_i\right)$$

$$= \frac{1}{n}\sum_{i=1}^{n} E(e_i)$$

$$= \frac{1}{n}\sum_{i=1}^{n} 0$$

$$= 0$$

$$V(\bar{Y}_n) =$$

$$V(\underline{\underline{\theta + b}} + \bar{e}_n)$$

$$= V(\bar{e}_n) = \frac{\sigma^2}{n}$$

$\bar{Y}_n$ has expected value $(\theta + b)$ and variance $\frac{\sigma^2}{n}$

$$V(e_1) = \sigma^2$$

$$V(e_i) = \sigma^2$$

$$V(\bar{e}_n) = \frac{\sigma^2}{n}$$

$$\theta$$

$$15.98 = 16.0 + 0 + (-0.02)$$

$$16.01 = 16.0 + 0 + (+0.01)$$

$$16.00 = 16.0 + 0 + (+0.00)$$

$$\vdots$$

$$15.97 = 16.0 + 0 + (-0.03)$$

Cancellation of $(+)$ and $(-)$

$$\bar{Y}_n = \theta + 0 + \frac{(+0.00) + (-.03) + (-0.02) + (+0.01)}{n}$$

$I_i \overset{IID}{\sim}$ mean $(\theta+b)$, variance $\sigma^2 < \infty$

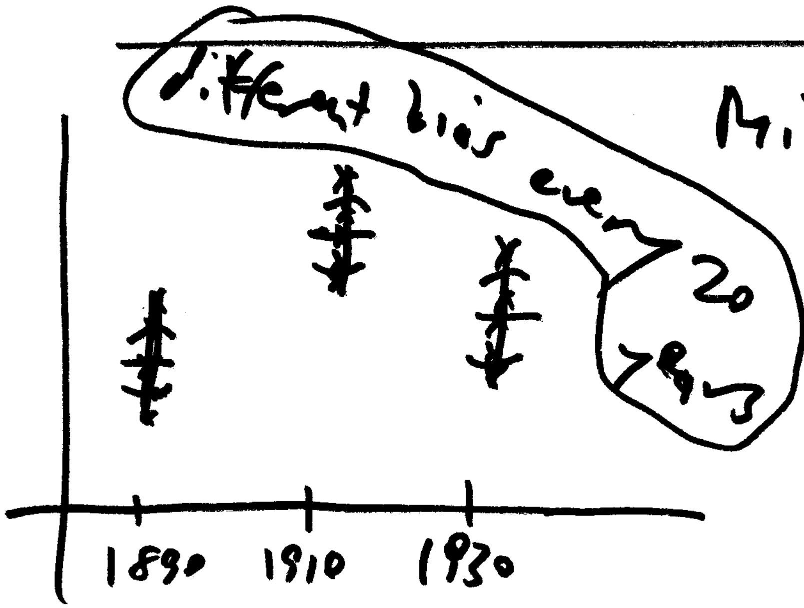
$\bar{I}_n \sim$ mean $(\theta + b)$, variance $\frac{\sigma^2}{n}$

$\boxed{\text{WLLN:}}$ $\bar{I}_n \xrightarrow{P} \theta + b \neq \theta$ unless $b = 0$

i.e., unless the measuring process is unbiased.

not automatically guaranteed for all measurement processes

Michaelson & Morley (speed of light in air) (1890)



$C \doteq 300$ Mm/sec

∴ $d = r \cdot t$