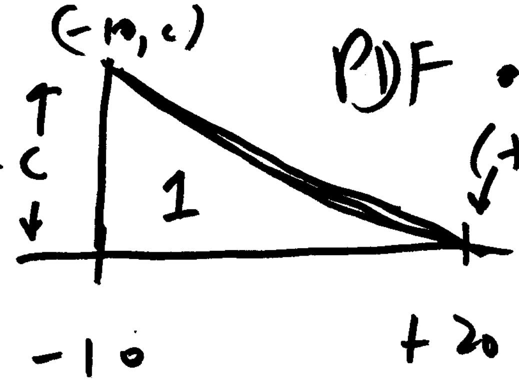
THT 1

5(c)(ii)



STAT 131

3 May 20

DJ extra office 1.5-hour session

$1 = \frac{1}{2}(30)(c) \rightarrow c = \frac{1}{15}$

1

to solve for

$v = -F_X^{-1}(\alpha)$

$\alpha = .01$

PDF

$$f_X(x) = \begin{cases} \frac{2}{45} - \frac{x}{450} & \text{for } x \in S_X \\ 0 & \text{else} \end{cases}$$

support of X

$S_X = (-10, +20)$

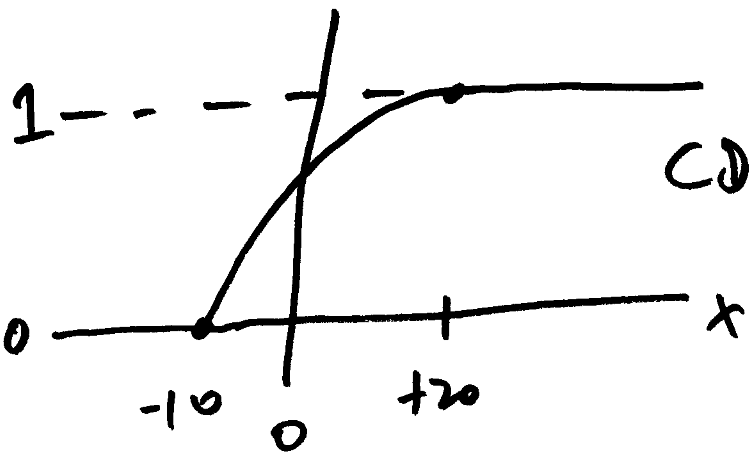
CDF of X $= F_X(x) = \begin{cases} \underline{\underline{0}} & \text{for } x < -10 \\ & -10 < x < 20 \\ 1 & \text{for } x > 20 \end{cases}$

for $-10 < x < +20$

$$F_X(x) = \int_{-10}^{x} \left(\frac{2}{45} - \frac{t}{450} \right) dt = \frac{-x^2 + 40x + 500}{900}$$

$$F_X(x_p) = \frac{-x_p^2 + 40x_p + 500}{900} = p$$

Wow



CDF $F_X(x)$

$$x = \begin{cases} 20 - 30\sqrt{1-p} \\ \text{or} \\ \cancel{30\sqrt{1-p} + 20} \end{cases}$$

wrong →

possible values of $-10 \le x \le +20$

possible values of $0 \le p \le 1$

$$F_X^{-1}(p) = 20 - 30\sqrt{1-p}$$

$$V = -F_X^{-1}(.01) = -\left[20 - 30\sqrt{.99}\right]$$

$= 8.85$ m

Alexis 4c(ii)

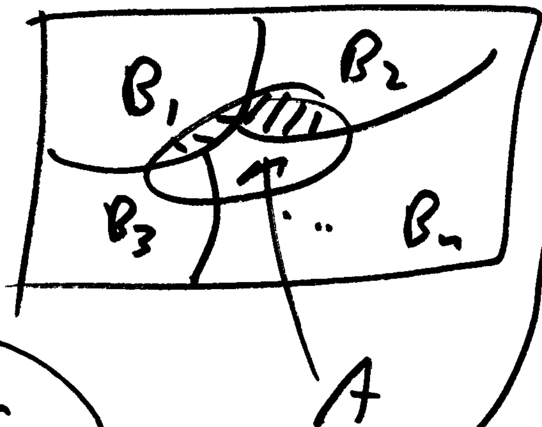
to show: for $r = 1, \ldots, n-1$

$$p_r = \frac{r}{n} \sum_{i=r+1}^{n} \frac{1}{i-1}$$

for fixed r $P(A) = P(A$ having pre-specified r before interviewing$)$

we already know: for $i > r$

$$P(A \mid B_i) = \frac{r}{i-1}$$



$P(A) =$
$P(A \text{ and } B_1) +$
$P(A \text{ and } B_2) +$
$\ldots + P(A \text{ and } B_n)$

$$P(A \text{ and } B_1) =$$

$$\cancel{P(A)\cdot P(B_1 \mid A)} \ \boxed{1}$$

$$= \underbrace{P(B_1)}_{\frac{1}{n}} \cdot P(A \mid B_1) \ \boxed{2}$$

$P(E \text{ and } F) =$
$P(E)\cdot P(F \mid E)$
$= P(F)\cdot P(E \mid F)$

B_i = best person is in slot i interview

$$P(A) = P(A \text{ and } B_1) + P(A \text{ and } B_2) + \ldots + P(A \text{ and } B_n)$$

$$= \underbrace{P(B_1)}_{\frac{1}{n}} \cancel{P(A \mid B_1)} + \underbrace{P(B_2)}_{\frac{1}{n}} \cancel{P(A \mid B_2)} + \ldots$$

$$+ \cancel{P(B_r) P(A \mid B_r)} + P(B_{r+1}) P(A \mid B_{r+1}) +$$

$$\dots + \overset{\frac{1}{n}}{p(B_n)}\, p(A \mid B_n)$$

we know that

$$p(A \mid B_i) = \begin{cases} 0 & \text{for } i \le r \\ \dfrac{r}{i-1} & i > r \end{cases}$$

$$p(A) = p_r$$

$$\frac{1}{n}\, p(A \mid B_{r+1}) + \frac{1}{n}\, p(A \mid B_{r+2}) + \dots + \frac{1}{n}\, p(A \mid B_n)$$

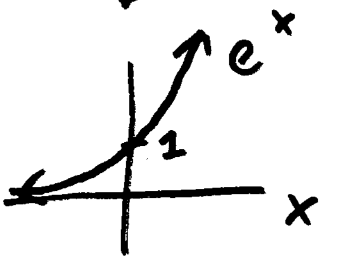
$$p_r = \frac{1}{n} \sum_{i=r+1}^{n} p(A \mid B_i) = \frac{1}{n} \sum_{i=r+1}^{n} \frac{r}{i-1} = \frac{r}{n} \sum_{i=r+1}^{n} \frac{1}{i-1}$$ ✓

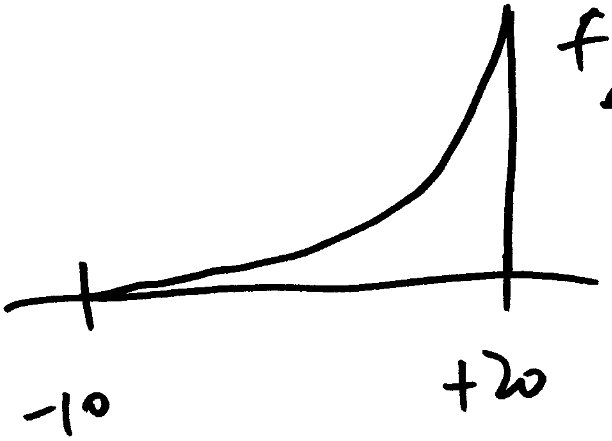
Isaac 2(d)

Ashvini 5(b)(iii)

$\doteq .0056$ $\quad \doteq 1.717$

$$f_X(x) = \begin{cases} c \exp\left(\dfrac{\lambda x}{10}\right) & \text{for } -10 \le x \le +20 \\ 0 & \text{else} \end{cases}$$





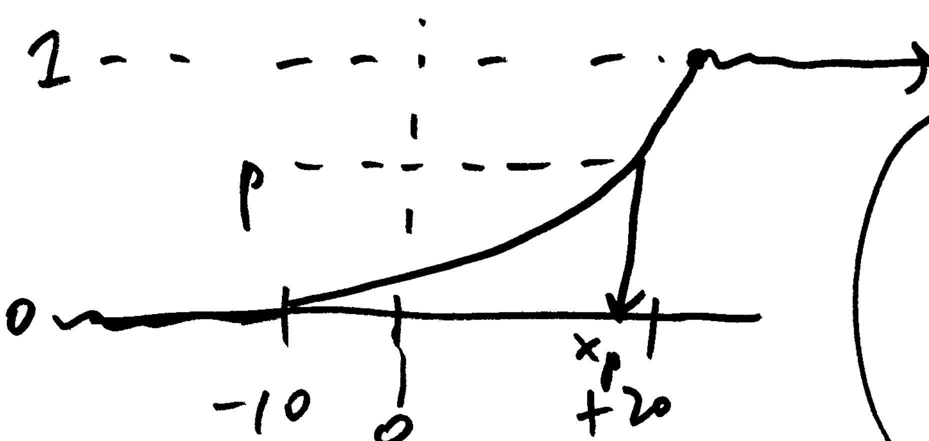
$f_X(x)$ PDF for X in 5(b)(iii)

1

2 CDF

we want $v = -F_X^{-1}(\alpha)$ ← .01

$$F_X(x) = \begin{cases} 0 & \text{for } x \le -10 \\ \int_{-10}^{x} f_X(t)\,dt & -10 \le x \le +20 \\ 1 & x \ge +20 \end{cases}$$



$$0.0324306 \cdot e^{0.171723 x_p} - .00582333 = p$$

for $-10 \le x_p \le +20$

solve for x_p

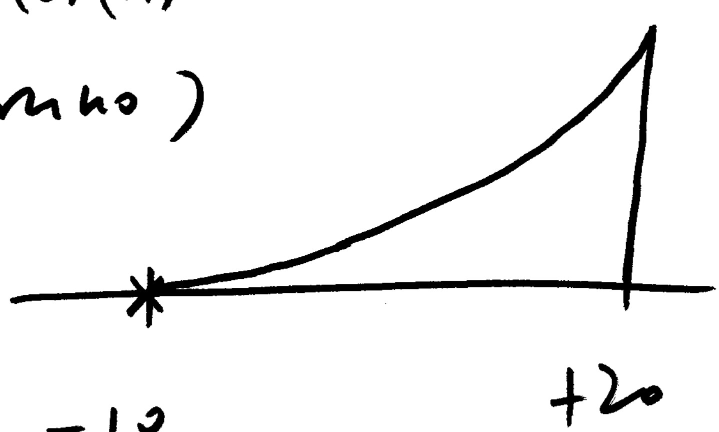
[3] inverse CDF

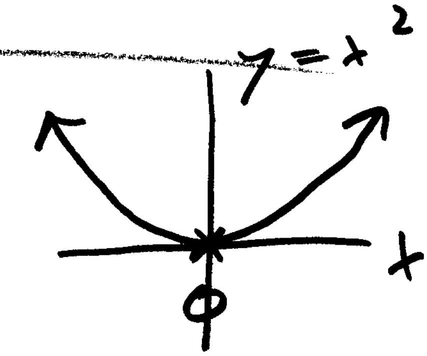
$$F_X^{-1}(p) = 5.82333 \log(30.8351\,p + 0.179563)$$

$$V = -F_X^{-1}(.01) \doteq \$4.18\,M$$

5(b)(ii)

(Bruno)





symmetric about its minimum

$$f_X(x) = \begin{cases} \underbrace{c}_{\frac{1}{9000}} \cdot (x+10)^2 & -10 \le x \le +20 \\ 0 & \text{else} \end{cases}$$

all PDFs

1. integrate to 1
2. ≥ 0

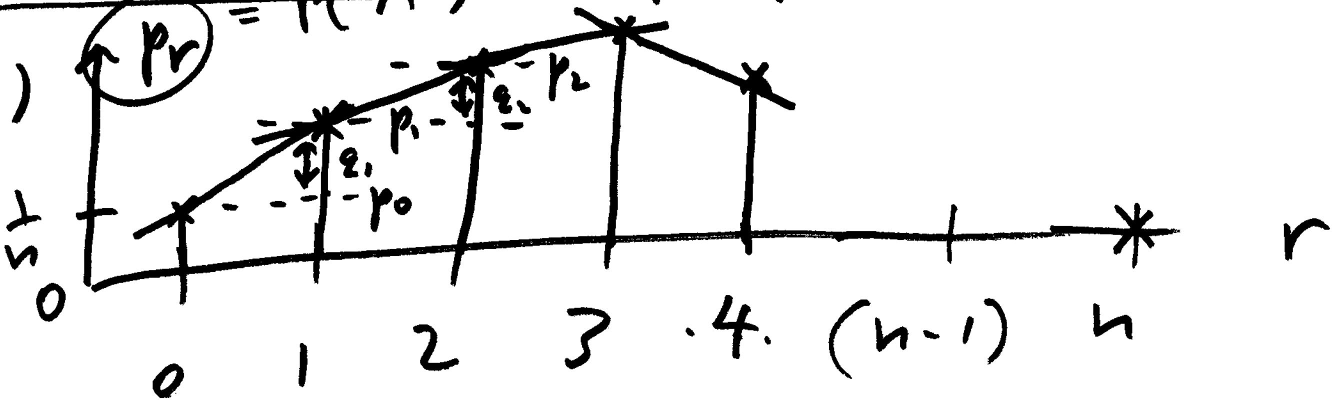
$$1 = \int_{-10}^{+20} c\,(x+10)^2\,dx$$

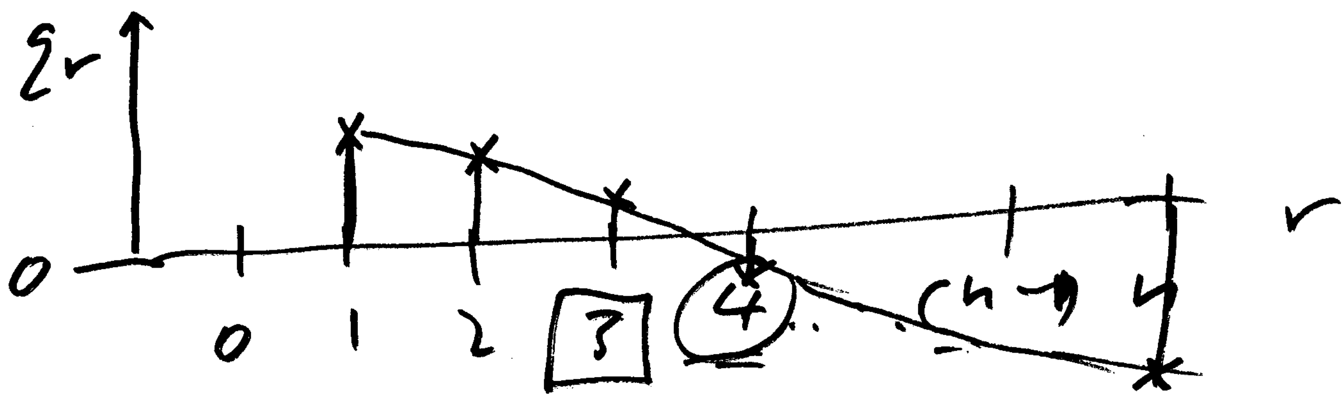
$$V = -F_X^{-1}(\alpha) \rightarrow -V = F_X^{-1}(\alpha)$$

$$F_X(-v) = F_X\left[\cancel{-}\, F_X^{-1}(\alpha)\right] = \alpha = .01$$

~~$p_r = p(A)$ for prespecified r~~

4(e)





$q_r = p_r - p_{r-1}$ for $r = 1, \dots, n$

$q_1 = p_1 - p_0$

$q_2 = p_2 - p_1$

~~lemma:~~

to show $q_n < 0$

$p_0 = p_0$

$p_1 = p_0 + \overset{(p_1 - p_0)}{q_1}$

$$p_2 = p_0 + \varepsilon_1 + \varepsilon_2$$

$$= \cancel{p_0} + (\cancel{p_1} - \cancel{p_0}) + (p_2 - \cancel{p_1})$$

$$\vdots$$

$$p_r = p_0 + \sum_{i=1}^{r} \varepsilon_i$$

let r^* be the largest r such that $\varepsilon_r > 0$

r^* is the best: $p_{r^*} > p_r$ for all other

discrete optimization

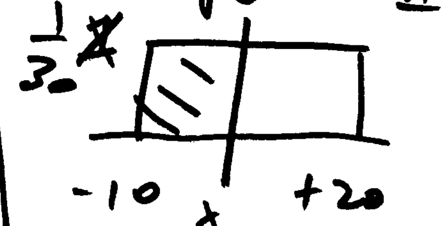
$$\int_{-10}^{x} \frac{1}{30}\, dt$$

#5)

PDF	VaR
5(b)(i)	\$7M
5(b)(ii)	?
5(b)(iii)	\$4.18M
5(c)(i)	~~\$9.85M~~ ?
5(c)(ii)	\$9.85M

5(c)(i)

PDF of X

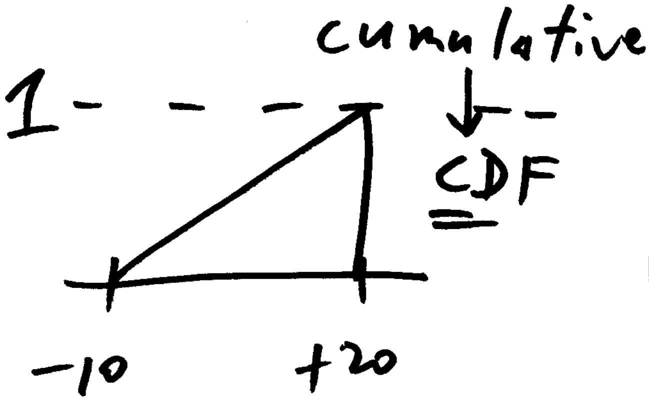


$1 = b \cdot h$

$= (30)h$

$h = \frac{1}{30}$

CDF $F_X(x) = \begin{cases} 0 & \text{for } x \le -10 \\ \int_{-10}^{x} \frac{1}{30}\, dt = \left(\frac{t}{30}\right)_{-10}^{x} = \frac{x+10}{30} & -10 \le x \le +20 \\ 1 & x \ge +20 \end{cases}$



$F_X(x) = P(X \le x)$