STAT 131
29 May 20

DD extra office 1.5-hour session (1)

$A > 0$ your stake, fixed in advance

you choose to bet $\$0 \le B \le \$A$; i.e., your action space is $\{B : 0 \le B \le A\}$

(bet nothing) (bet it all)

(uncountably infinite action space)

you win bet with (known) probability $0 < p < 1$

X = value of stake after gamble

if win, stake goes from A to $(A + B)$

lose $(A - B)$

as $p \uparrow$, optimal $B \uparrow$

②

(PMF of X) (for fixed A, B) (a)

$$f_X(x) = \begin{cases} p & \text{if } x = A+B \\ (1-p) & \underline{\underline{A-B}} \\ 0 & \text{else} \end{cases}$$

$U_1(X) = X$

$U_2(X) = 1 + \log(X)$

case I : $U_1(X) = X$

$$E[U_1(X)] = E(X)$$

$$= (A+B)\,p + (A-B)(1-p)$$

$$= \cancel{Ap} + Bp + A - \cancel{Ap} - B + Bp$$

$$= A + 2Bp - B = B(2p-1) + A$$

~~holding~~ (A, p) constant this is linear in B

optimal B^* maximize $E[U_1(X)]$

usual method: set $\frac{d}{dB} E[u_1(B)] = 0$ & solve for B

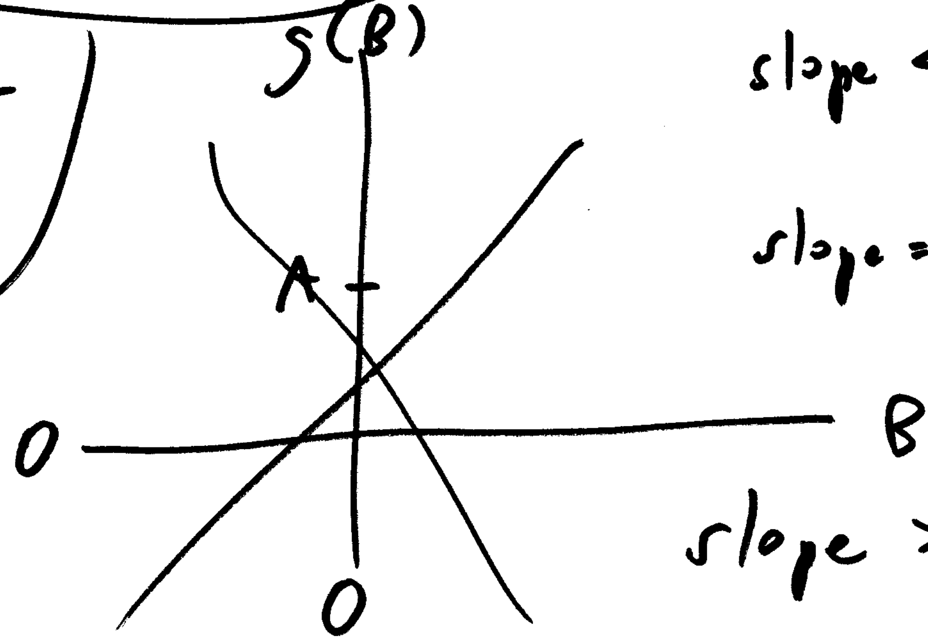
$E[u_1(B)] = A + B(2p-1) = g(B)$

$\frac{d}{dB}[A + B(2p-1)] = 2p-1$ (independent of B (!))

$= 0$ iff $p = \frac{1}{2}$

$g(B) = A + B(\underline{2p-1})$

y-intercept A



slope < 0 iff $p < \frac{1}{2}$

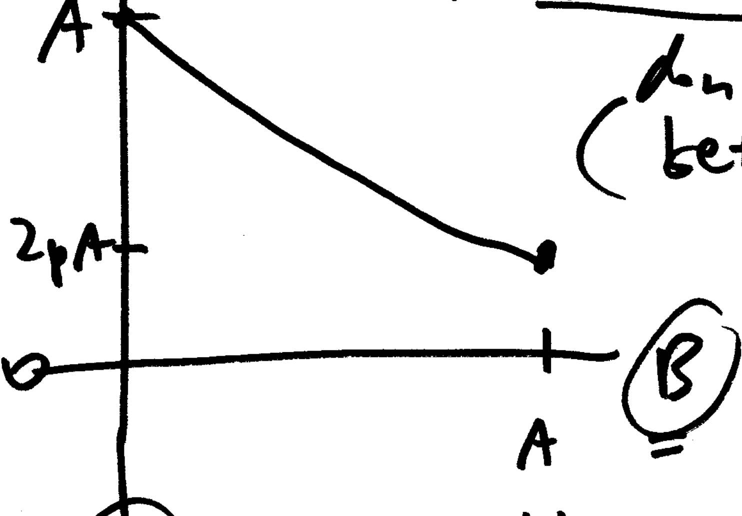
slope $= 0$ iff $p = \frac{1}{2}$

slope > 0 iff $p > \frac{1}{2}$

$p < \frac{1}{2}$

$\boxed{E[u(X)]}$ $\boxed{B^* = 0}$

(don't bet)



0 reasonable because you expect to lose

feasible B: ④

$0 \leq B \leq A$

$g(B) = A + B(2p-1)$

$g(A) = A + A(2p-1)$

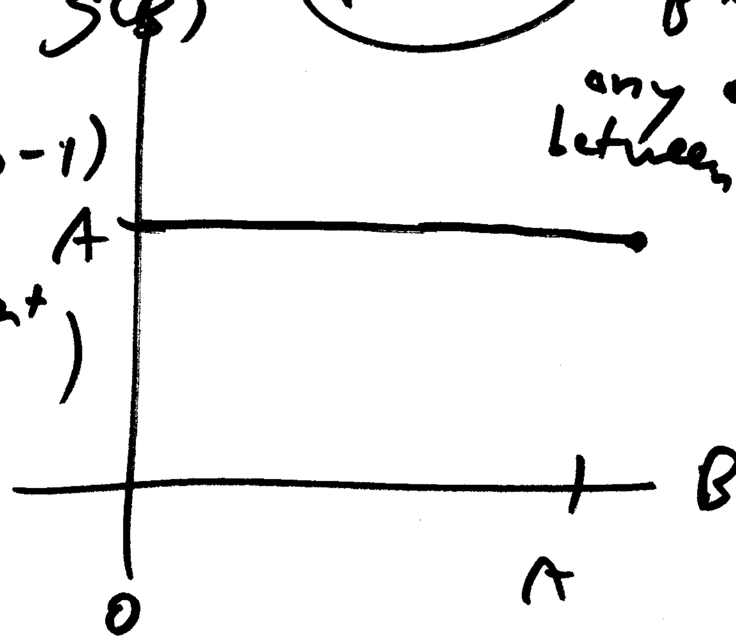
$= A + 2pA - A$

$= 2pA$

$p = \frac{1}{2}$

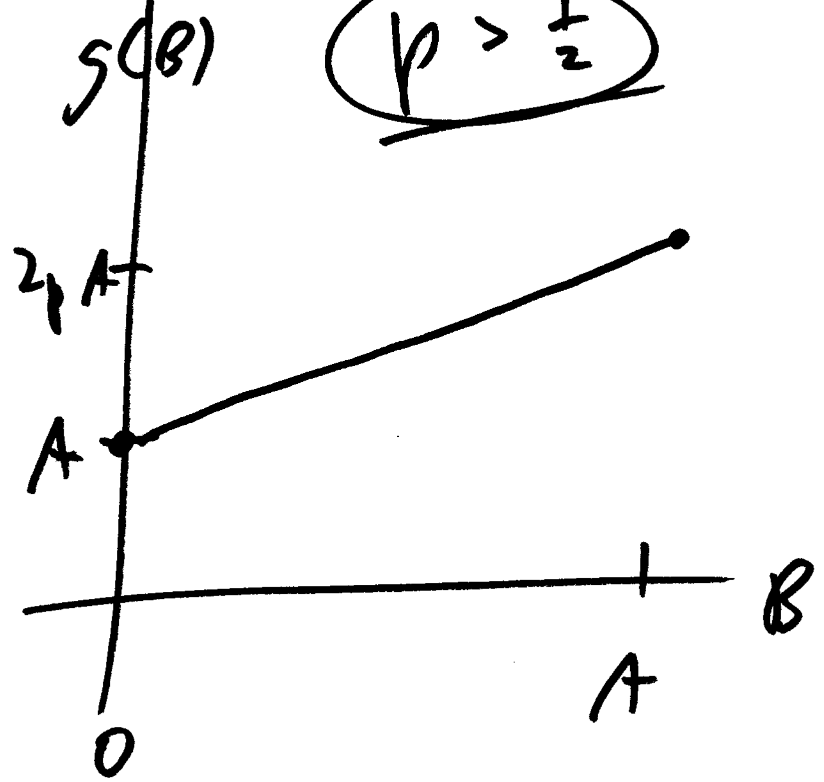
$g(B) = A + B(2p-1)$

$= A$ (constant in B)



$B^* =$ any amount between 0 and A

$p > \frac{1}{2}$



$g(B) = A + B(2p-1)$

$g(A) = 2pA$

$B^* = A$ (bet it all)

now, instead,

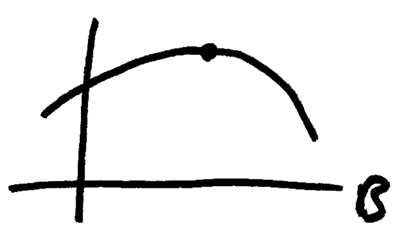
$U_2(X) = 1 + \log(X)$

$E[U_2(X)] = E[1 + \log(X)]$

$= 1 + E[\log(X)]$

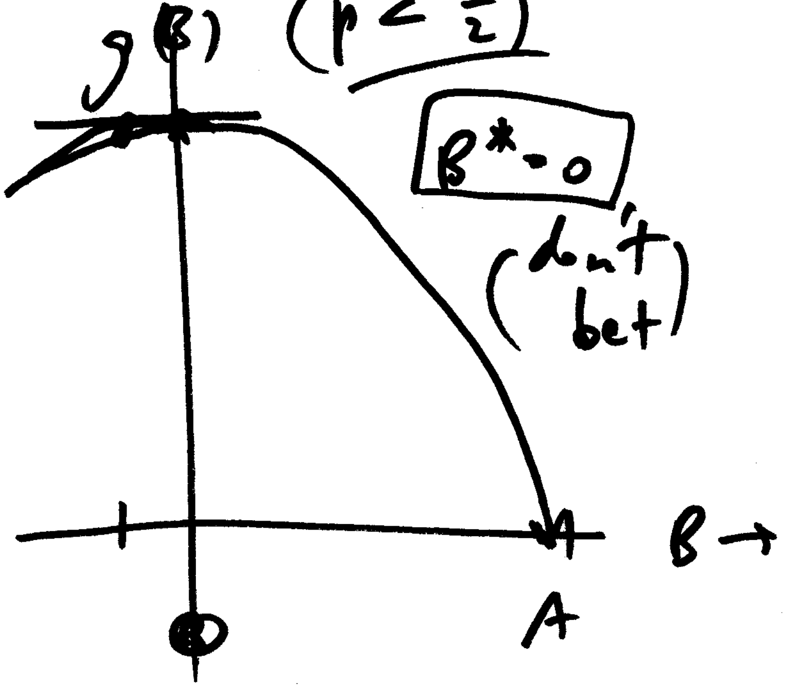
LOTUS

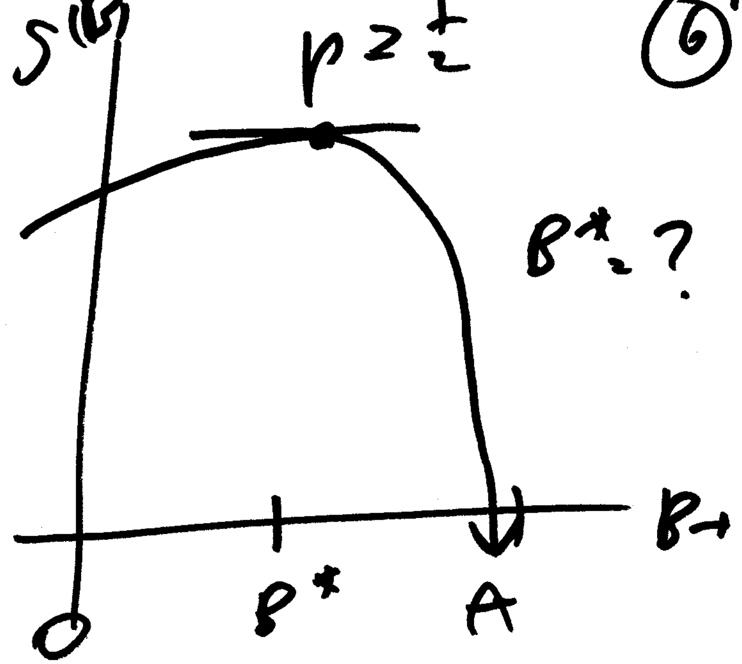
$= 1 + \log(A+B) \cdot p + \log(A-B)(1-p)$



we want concave

(6)





$$\frac{d}{dB} E\left(u_2(X)\right) = \frac{A(2p-1) - B}{(A-B)(A+B)} = 0$$

iff $B = B^* = A(2p-1) = \underline{2A(p - \tfrac{1}{2})}$