STAT 131
29 Apr 20
DD office
1.5 hr
session

real purpose of education:
to learn how to learn
(to learn how to teach
yourself new ideas & methods) ①

meta-code (process of problem-solving)

when you encounter a new problem $\mathbb{P}_{new}$, it's often helpful to find another problem $\mathbb{P}_{old}$ with 2 properties:

① $\mathbb{P}_{old}$, $\mathbb{P}_{new}$ similar in (all) relevant ways

② you know how to solve $\mathbb{P}_{old}$:

then you can use solution to $\mathbb{P}_{old}$

to help in solving $\mathbb{P}_{new}$.

THT 1

2(a)

$\mathbb{P}_{new}$

$\mathbb{P}_{old}$

T-S case study

at least one

$P($1 or more T-S babies in family of $n = 5$ kids, both parents carriers$)$

✓ I ? I D ? ✓

$[\, P(\text{T-S on any 1 kid}) = p = \frac{1}{4} \,)$

grand prize)

2(a) $P($at least 1 GP among 182,900,000 → n lottery participants,

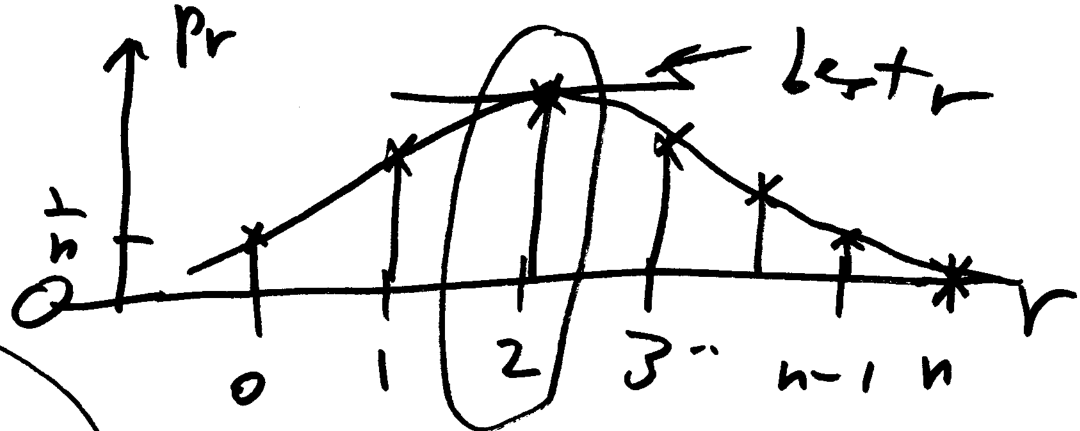
$P(GP) = p = \dfrac{1}{292{,}201{,}338}$

✓ I ? I ? D ✓

$P($at least one success (S) in n IID success-failure trials,

$P(S \text{ on any 1 trial}) = p\,) = 1 - (1-p)^n$

THT 1 #4

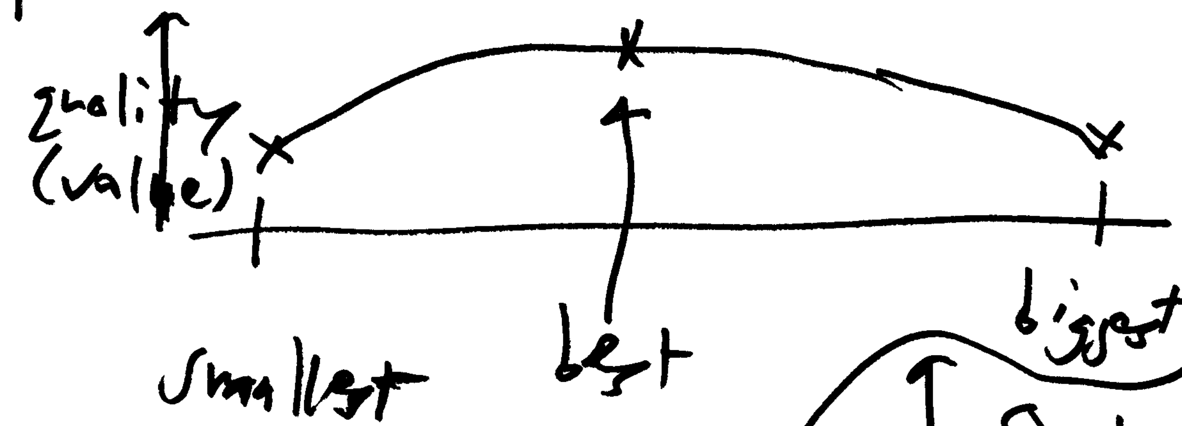


n candidates

r size of quality pool QP

p_r = P(we hire best person with quality pool of size r)

r ↑ we get more & better info about quality of applicants, but as r ↑ our chance of not hiring best person because that person is in QP ↑



we choose a point on this line

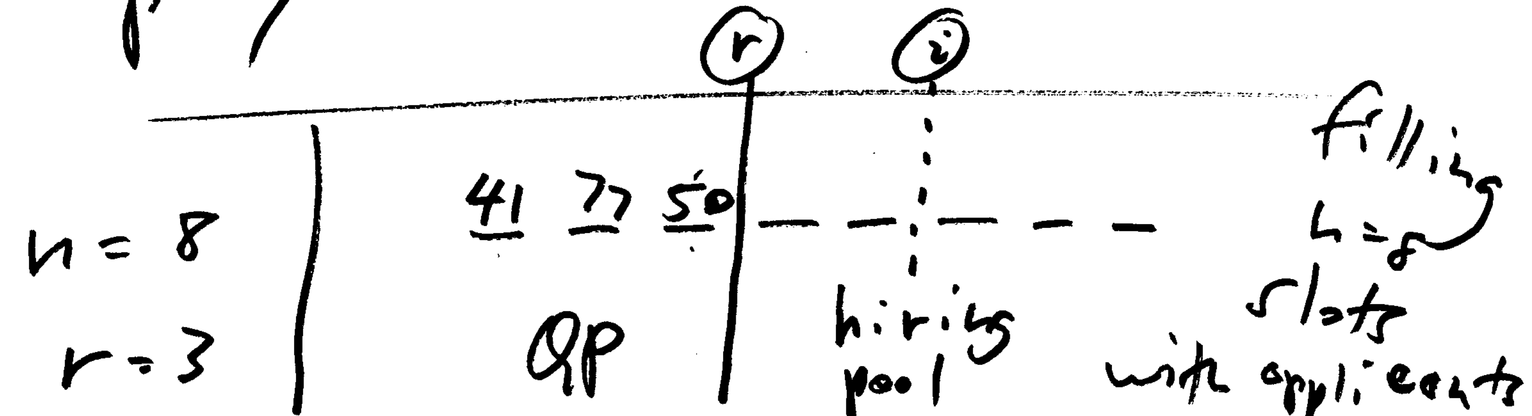
meta-code (process of problem-solving)

when the problem of interest is too abstract to see useful patterns,

general
$1 \le n$
$0 \le r < n$

get playful: pick some specific numbers &

play around with result



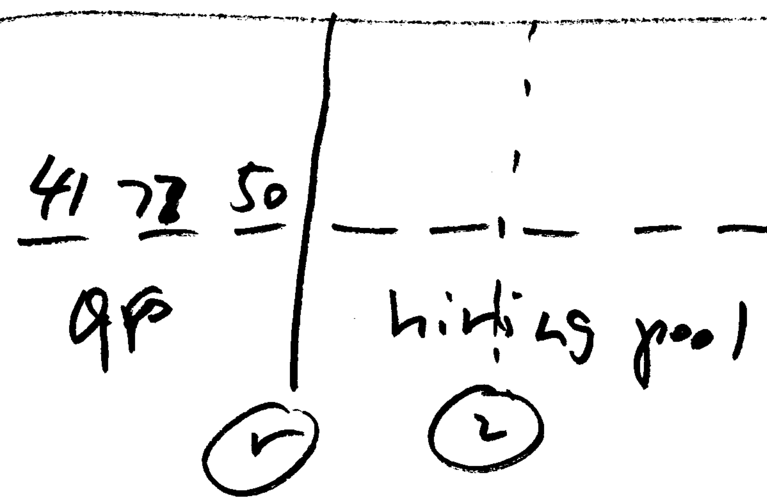
assume no ties in quality assessment

invent quality score, from 0 - 100

pick $\underline{i > r}$; ex. $i = 5$

\# slots best person \underline{could} land in $= i = 5$

ELM? yes



4(a) prob $= \dfrac{r}{i} = \dfrac{3}{5}$

4(b) $P(A \mid B_i) = 0$ for $i \le r$

we hire best

best person is in slot i

p_r = P(A having pre-specified r before interviewing begins) $0 \le r \le n$

we hire best

$$p_0 = \frac{1}{n} \quad ✓$$

$0 < r < n$

show

$$p_r = \frac{r}{n} \sum_{i=r+1}^{n} \frac{1}{i-1}$$

we know

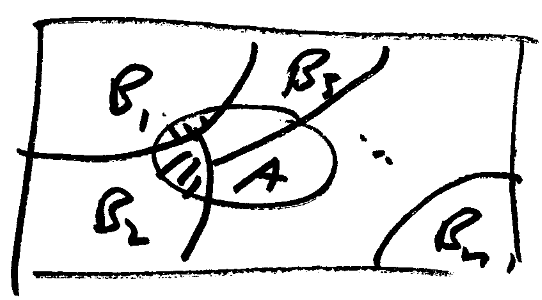
$$P(A \mid B_i) = \begin{cases} 0 & \text{for } i \le r \\ \dfrac{r}{i-1} & \text{for } i > r \end{cases}$$

$$P(B_i) = \frac{1}{n}$$

we want

P(A) hard

P(A | B_i) easy



$$P(A) = P\left[(A \text{ and } B_1) \text{ or } (A \text{ and } B_2) \text{ or } \dots \text{ or } (A \text{ and } B_n) \right]$$

B_i mutually exclusive (partition)

$$= P(A \text{ and } B_1) + P(A \text{ and } B_2) + \dots + P(A \text{ and } B_n)$$

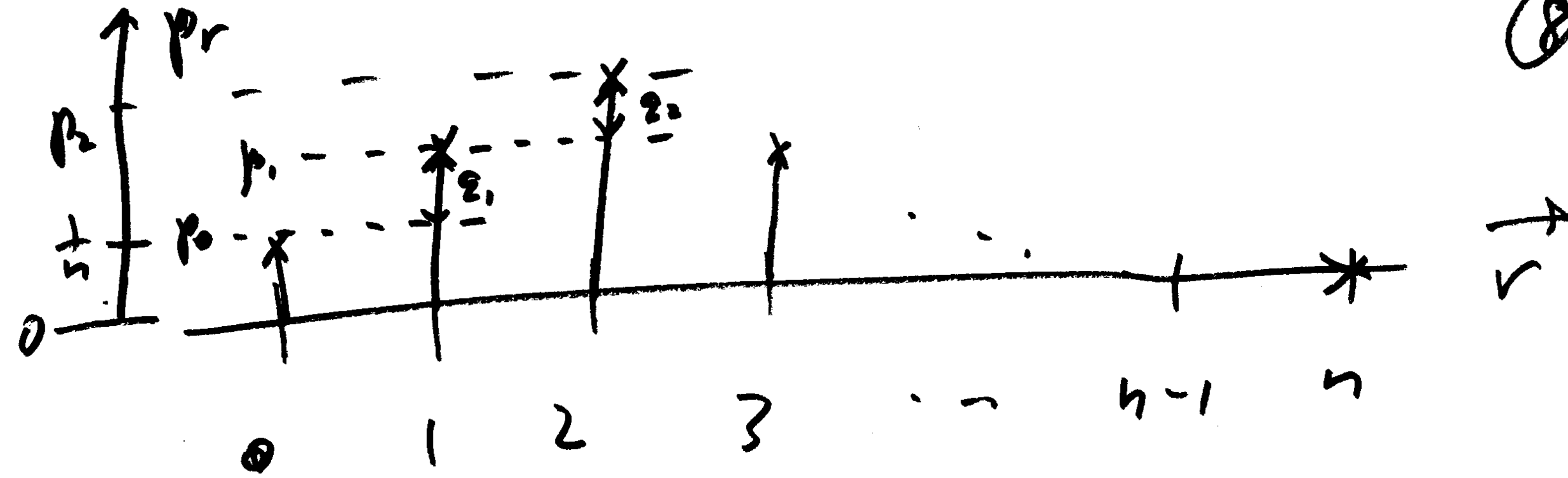
$$= \underbrace{P(B_1)}_{\frac{1}{n}} P(A \mid B_1) + P(B_2) P(A \mid B_2) + \dots + P(B_n) P(A \mid B_n)$$

$$P(B_i) = \frac{1}{n}$$

$$P(A \mid B_i) = \begin{cases} 0 & \text{for } i \le r \\ \dfrac{r}{i-1} & i > r \end{cases}$$

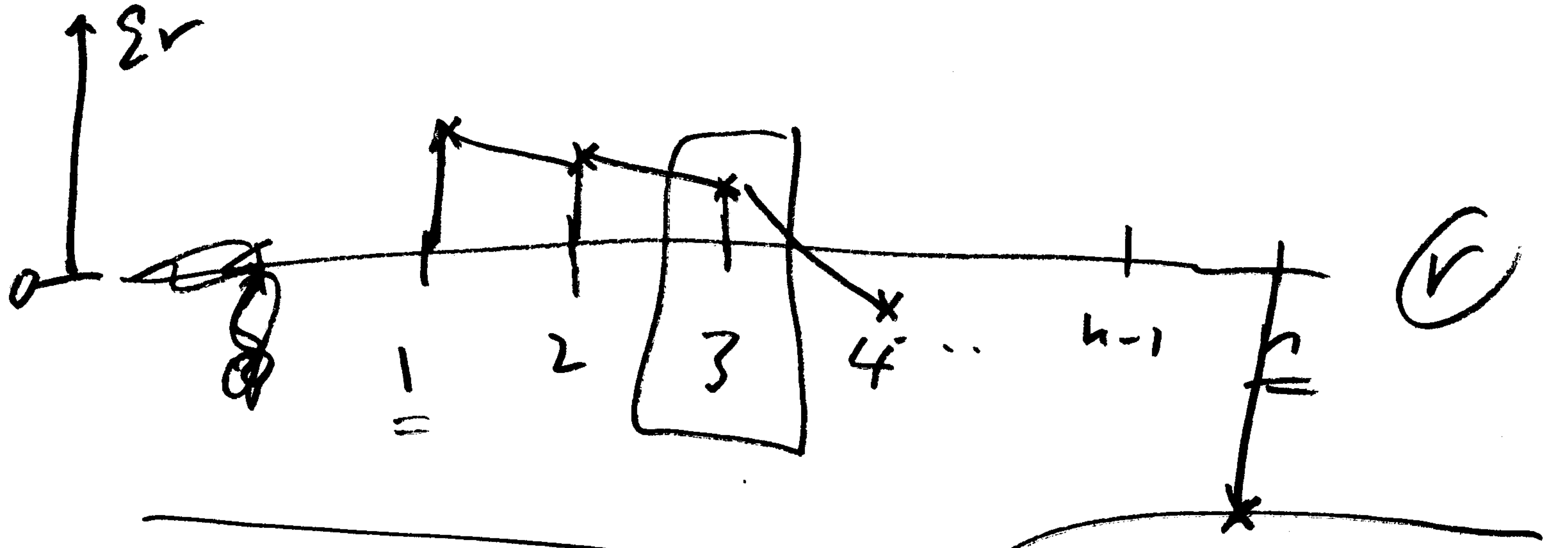
$$= P(B_{r+1}) P(A \mid B_{r+1}) + \dots + P(B_n) P(A \mid B_n)$$

$$q_r = (p_r - p_{r-1}) \quad \text{for} \quad 1 \le r \le n$$



$$\varepsilon_1 = (p_1 - p_0) \qquad \varepsilon_2 = (p_2 - p_1)$$

$$\varepsilon_r = p_r - p_{r-1}$$



$$\varepsilon_1 = p_1 - p_0 = \left(\frac{1}{n}\sum_{i=2}^{n}\frac{1}{i-1}\right) - \frac{1}{n}$$

$r = 1, 2, \dots, n-1$

$$p_r = \frac{r}{n}\sum_{i=r+1}^{n}\frac{1}{i-1}$$

$$\xi_2 = \mu_2 - \mu_1$$

$$= \left(\frac{2}{n}\sum_{i=3}^{n}\frac{1}{i-1}\right) - \left(\frac{1}{n}\sum_{i=2}^{n}\frac{1}{i-1}\right)$$

$$\sum_{i=3}^{n}\frac{1}{i-1} = \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n-1}$$

$$\sum_{i=2}^{n}\frac{1}{i-1} = 1 + \frac{1}{2} + \dots + \frac{1}{n-1}$$

$$\sum_{i=1}^{\infty}\frac{1}{i} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

$$= +\infty$$