STAT 131
28 Apr 20
DD office 1.5 hour session

(haphazardly chosen person) $\leftrightarrow$ one person chosen in a like-at-random manner

①

choose 1 person at random from {all attendees} $\rightarrow$ ELM applies ✓

$P(E) = 0.9$, etc

(relative frequency (90%)) $\rightarrow$ probability

$$\underbrace{p}_{\text{unknown}} = P(St \mid \overbrace{Sh}^{\text{data}})$$

$0.8 = P(Sh \mid St)$

$P(\text{data} \mid \text{unknown})$

A | $p_A = P(A)$

$o_A = \dfrac{p_A}{1 - p_A}$

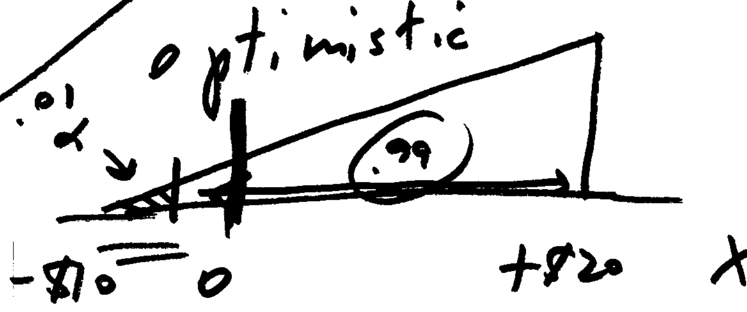
odds ratio in favor of A being true $= \dfrac{P(A)}{P(\text{not} A)}$

$$\underbrace{\left[\frac{P(st \mid sh)}{P(E \mid sh)}\right]}_{o_{st|sh}} = \left[\frac{\overset{0.1}{P(st)}}{\underset{0.9}{P(E)}}\right] \cdot \left[\frac{\overset{0.8}{P(sh \mid st)}}{\underset{0.15}{P(sh \mid E)}}\right]$$

$$= \left(\frac{0.1}{0.9}\right)\left(\frac{0.8}{.15}\right)$$

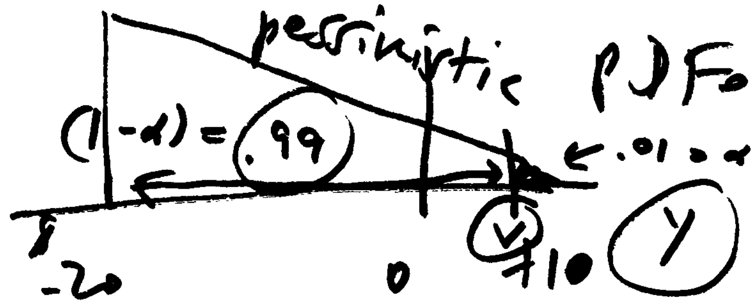
THT 1 AS

X = change in value



PDF of X

$Y = -X$

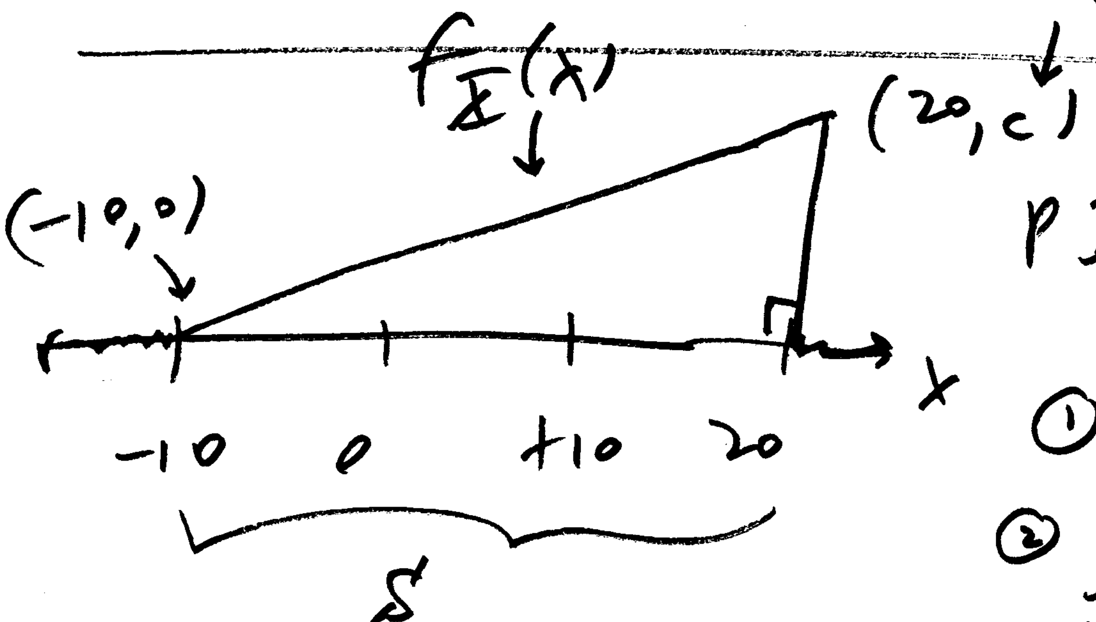


α small positive number (ex. $\alpha = 0.01$

ex. 0.99

VaR = $\underline{(1-\alpha)}$ quantile of dist. of Y

$f_X(x)$ linear (+ slope), goes through $(-10, 0)$, $(20, c)$



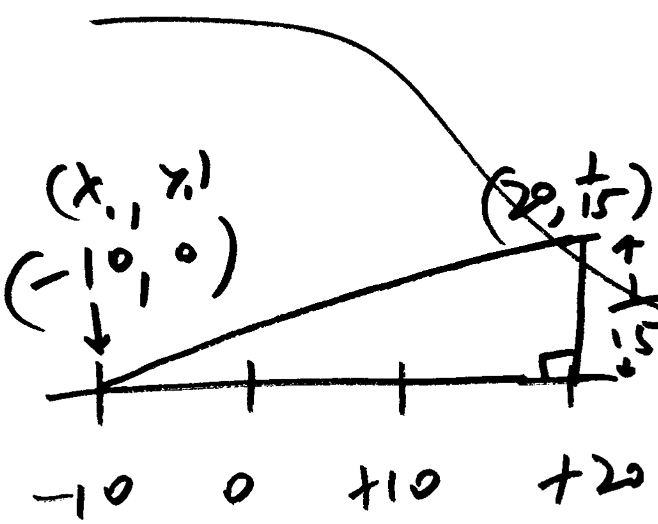
PDF of X

① $f_X(x) \geq 0$

② $\int_{-10}^{+20} f_X(x) = 1$

area of ◺ is $\frac{1}{2}(\text{base})(\text{height})$

$$\frac{1}{2}\underbrace{\left(20-(-10)\right)}_{30}\cdot c = 1$$



$$c = \frac{1}{15}$$

$\boxed{1} \to$ PDF of X

$$f_X(x) = \begin{cases} \dfrac{x+10}{450} & \text{for } -10 \le x < +20 \\ 0 & \text{else} \end{cases}$$

$$\frac{y - y_1}{x - x_1} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$$

$$\frac{y - 0}{\cancel{x - (-10)}} = \frac{\frac{1}{15} - 0}{20 - (-10)}(x + 10)$$

$$y = \frac{1}{450}(x + 10)$$

$$VaR = v = -F_X^{-1}(\alpha)$$

$\boxed{2}$ CDF of X

$\boxed{3}$ inverse CDF (quantile function) of X

$$f_X(x) = \begin{cases} \frac{x+10}{450} & \text{for } -10 < x < 20 \\ 0 & \text{else} \end{cases}$$

for $-10 < x < +20$

$$\frac{t}{450} + \frac{1}{45}$$

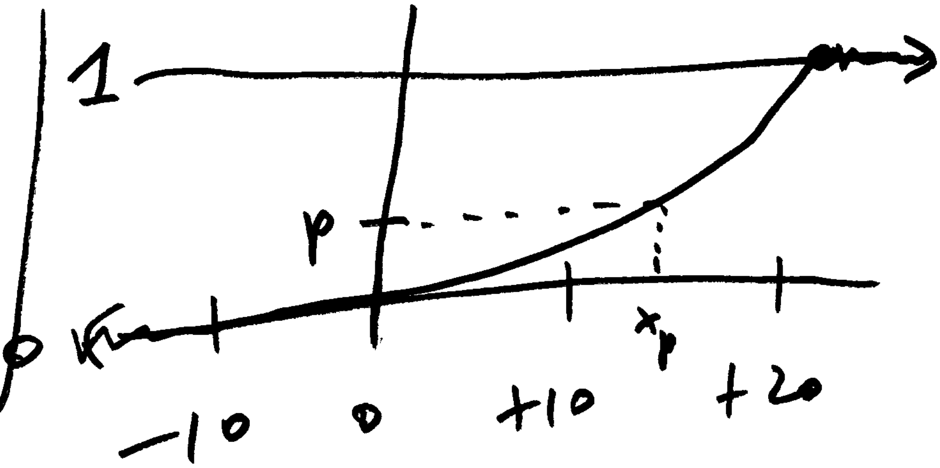
$$F_X(x) = \int_{-10}^{x} \frac{t+10}{450}\, dt$$

$$= \left[\frac{(t+10)^2}{2 \cdot 450} \right]_{-10}^{x}$$

$$= \frac{1}{900} (x+10)^2 - 0$$

$$F_X(x) = \begin{cases} 0 & \text{for } x < -10 \\ (x+10)^2/900 & -10 < x < 20 \\ 1 & x > +20 \end{cases}$$

sketch of $F_X(x)$



$$\boxed{\frac{(x_p+10)^2}{900} = p} = F_X(x_p)$$

$$(x_p+10)^2 = 900p$$

$$x_p + 10 = 30\sqrt{p}$$

$$x_p = \boxed{30\sqrt{p} - 10 = F_X^{-1}(p)}$$

$\alpha = .01$

VaR with this PDF for X $= -F_X^{-1}(\alpha) = -(30\sqrt{\alpha} - 10)$

$$= 10 - 30\sqrt{\alpha}$$

$$= \$7\text{M}$$