STAT 131
27 May 20

DD extra office 1.5-hour session

$$E(\,Y \mid X = x\,) = \frac{p(x)}{p(x)+p(\frac{x}{2})}\,(2x) + \frac{p(\frac{x}{2})}{p(x)+p(\frac{x}{2})}\left(\frac{x}{2}\right)$$

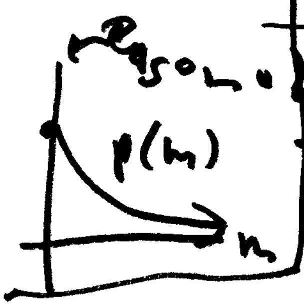
your \$ if you trade

claim: $\frac{1}{2}$ $\qquad$ $\frac{1}{2}$

1

is there any prior $p(m)$ such that

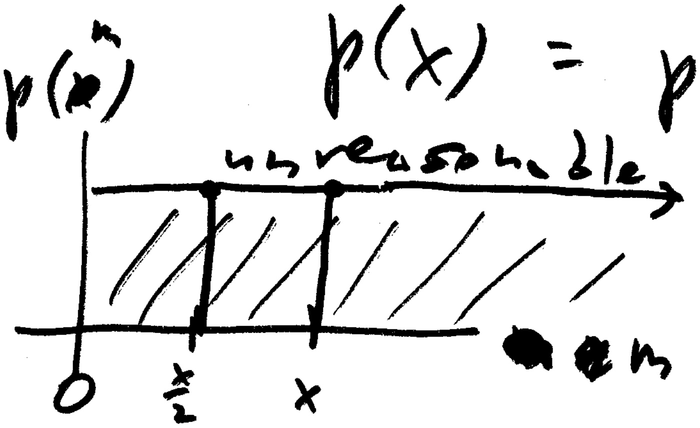
$$\boxed{\frac{p(x)}{p(x)+p(\frac{x}{2})} = \frac{1}{2}}$$ for all $x > 0$!



$$2\,p(x) = p(x) + p\left(\frac{x}{2}\right)$$

Confusion between $E(Y)$ and $E(Y \mid X = x)$

$$p(x) = p\left(\frac{x}{2}\right) \text{ for all } x > 0$$



improper prior: integrates to ∞

THT 3(b)(i)

$(Y \mid \lambda) \sim \text{Poisson}(\lambda)$

\# of interesting events in $[0, T]$

$(\lambda > 0)$

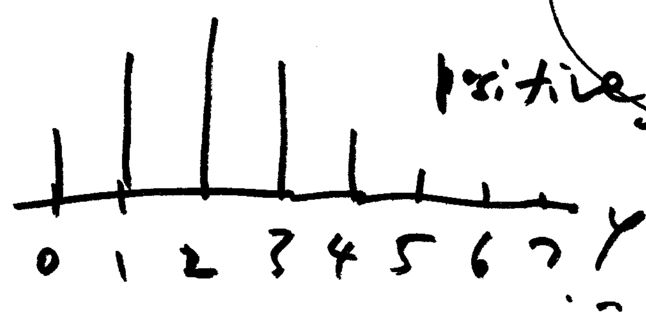
PMF

$$f_Y(y) = P(Y = y \mid \lambda) = \begin{cases} \dfrac{\lambda^y e^{-\lambda}}{y!} & \text{for } y = 0, 1, \ldots \\ 0 & \text{else} \end{cases}$$

S_Y

~~$P(Y = y \mid \lambda)$~~

positive skewness



$$\psi_{(Y \mid \lambda)}(t) = E(e^{tY} \mid \lambda)$$

(t any real #)

(LOTUS)

$$= \sum_{y=0}^{\infty} e^{ty} \cdot P(Y = y \mid \lambda)$$

$$= \sum_{y=0}^{\infty} e^{ty} \cdot \frac{\lambda^y e^{-\lambda}}{y!} \quad e^{\lambda(e^t - 1)}$$

③

$$\psi_{(Y|\lambda)}(t) = \sum_{y=0}^{\infty} \frac{e^{ty} \cdot \lambda^{y} \cdot e^{-\lambda}}{y!}$$

$$= e^{-\lambda} \sum_{y=0}^{\infty} \frac{e^{ty} \lambda^{y}}{y!}$$

$$= e^{-\lambda} \sum_{y=0}^{\infty} \frac{(\lambda e^{t})^{y}}{y!}$$

$$= e^{-\lambda} e^{\lambda e^{t}} = e^{-\lambda + \lambda e^{t}}$$

$$= e^{\lambda(e^{t}-1)} \quad \checkmark$$

for all $x \in \mathbb{R}$

$$e^{x} = \sum_{n=0}^{\infty} \frac{x^{n}}{n!}$$

$n \leftrightarrow y$

$x \leftrightarrow \lambda e^{t}$

$$E(Y|\lambda) = \left[\frac{d}{dt} \psi_{(Y|\lambda)}(t) \right]_{t=0}$$

$$= \lambda$$

$$V(Y|\lambda) = \lambda$$

$$\text{skewness}(Y \mid \lambda) = \frac{1}{\sqrt{\lambda}} > 0 \quad \left(\begin{array}{c}\text{long} \\ \text{right-} \\ \text{hand} \\ \text{tail}\end{array}\right)$$

$$(\lambda > 0) \quad = \underline{\underline{O}}\left(\frac{1}{\sqrt{\lambda}}\right)$$

$$\to 0 \text{ as } \lambda \uparrow \infty$$

~~dist.~~	~~mean~~	~~variance~~	~~SD~~	~~skewness~~
Poisson(λ)	λ	λ	$\sqrt{\lambda}$	$\frac{1}{\sqrt{\lambda}}$

cont.

THT 2
4(c)(i)

$$(W \mid \alpha, \beta) \sim \text{Pareto}(\alpha, \beta)$$

$$\overset{\text{PDF}}{\leftrightarrow} f_W(w \mid \alpha, \beta) = \quad \begin{pmatrix}\alpha > 0 \\ \beta > 0\end{pmatrix}$$

normalizing constant

$$\alpha \beta^{\alpha} \, w^{-(\alpha+1)} \, I(w \geq \beta)$$

$$\ell(\theta \mid \underset{\sim}{y}) = c \cdot \theta^{-n} \, I(\theta \geq m)$$

$$(y_1, \dots, y_n) \quad \boxed{\begin{array}{c} -n = \\ -(\alpha+1) \\ \hline \alpha = n-1 \end{array}} \quad \max(\underset{\sim}{x})$$

solve for α

ℓ	Pareto
$n-1$	α
m	β

Bayes's theorem for T/F statements

$\mathcal{U}$ = unknown

$\mathcal{D}$ = data

prior info. about $\mathcal{U}$

$$P(\mathcal{U} \mid \mathcal{D}) = \frac{P(\mathcal{U})\, P(\mathcal{D} \mid \mathcal{U})}{P(\mathcal{D})}$$

posterior information about $\mathcal{U}$ given $\mathcal{D}$

normalizing constant

likelihood info. about $\mathcal{U}$ given $\mathcal{D}$

cont. r.v.

unknown = $\theta > 0$

data = y

(this works)

prior PDF

unnormalized likelihood PDF

posterior PDF

$$f_{(\Theta \mid y)}(\theta \mid y) = \frac{f_{\Theta}(\theta)\, \ell(\theta \mid y)}{\text{normalizing constant}}$$

$$f_{(\Theta|y)}(\theta|y) = c \cdot \boxed{f_{\Theta}(\theta)} \cdot \ell(\theta|y)$$

$\begin{pmatrix}\alpha>0\\ \beta>0\end{pmatrix}$

Pareto(α, β) · Pareto$(n-1, m)$

$$= c \cdot \boxed{c\,\theta^{-(\alpha+1)}\, I(\theta \geq \beta)} \cdot \boxed{c\,\theta^{-n}\, I(\theta \geq m)}$$

missing step: once y ~~has arrived~~

~~starts as a~~ f'n of y for fixed θ

$$\boxed{f_{(\Theta|y)}(\theta|y)} = \boxed{c \cdot f_{\Theta}(\theta) \cdot c \cdot f_{(y|\theta)}(y|\theta)}$$

we want to treat this as a f'n of θ for fixed y

set equal

$\ell(\theta|y)$

need to think of this as a f'n of θ for fixed y