THT2 #1

STAT 131
24 May 20

before the game begins, you have uncertainty about the following:

m = amount referee puts into E1 (envelope) (m)

X = amount you see in your E (x)

Y = amount your opponent sees in her/his/their E (y)

$$E = \begin{cases} 1 & \text{if you got E1} \\ 2 & \text{E2} \end{cases}$$

DD extra office 1.5-hour session

(hard) ①

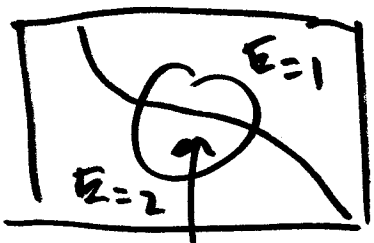
$P(Y = y \mid X = x) = ?$ ⊛

let's so partition over another unknown such that, if it were known, ⊛ would be easier

partitioning over m is messy & difficult (try it)

so let's partition over $E = 1$ or 2

$P(Y = y \mid X = x)$



$(Y = y \mid X = x)$

$$= P(Y = y, E = 1 \mid X = x) + P(Y = y, E = 2 \mid X = x)$$

$$= P(Y = y \mid E = 1, X = x) \cdot P(E = 1 \mid X = x) + P(Y = y \mid E = 2, X = x) \cdot P(E = 2 \mid X = x)$$

and now, several excellent things happen: ②

① given $X = x$, $(E=1) = (M = x)$ (why?)

and $(E=2) = (M = \frac{x}{2})$ (why?) ② given $(E=1, X=x)$

the only possible value for Y is $y = 2x$ (why?)

and given $(E=2, X=x)$ the only possible value for Y is $y = \frac{x}{2}$ (why?) so the conditional

distribution $P(Y=y \mid X=x)$ looks like this:

Fix an $x \in S_X$ the support of $(Y \mid X=x)$ is just the 2 points $\{2x, \frac{x}{2}\}$ (from ② above)

so $P(Y=2x \mid X=x) = \boxed{\cancel{P(Y=2x \mid E=1, X=x)}}^{1} \cdot P(M=x \mid X=x)$

$+ \boxed{\cancel{P(Y=2x \mid E=2, X=x)}}^{0} \cdot P(M=\frac{x}{2} \mid X=x)$

$= P(M=x \mid X=x)$

$$P(Z = \tfrac{x}{2} \mid X = x) = \frac{\cancelto{0}{\boxed{P(Z = \tfrac{x}{2} \mid E = 1, X = x)}}}{P(M = x \mid X = x)} \quad ③$$

$$+ \cancelto{1}{\boxed{P(Z = \tfrac{x}{2} \mid E = 2, X = x)}} \cdot P(M = \tfrac{x}{2} \mid X = x)$$

$$= P(M = \tfrac{x}{2} \mid X = x)$$

$$= \frac{p(\frac{x}{2})}{p(x) + p(\frac{x}{2})}$$

and

$$P(Z = 2x \mid \underline{X = x}) = P(M = x \mid X = x)$$

$$= \frac{p(x)}{p(x) + p(\frac{x}{2})}$$

fix $x \in S_X$

$$f_{Z|X}(y \mid x) = \begin{cases} \dfrac{p(x)}{p(x) + p(\frac{x}{2})} & \text{if } y = 2x \\[2ex] \dfrac{p(\frac{x}{2})}{p(x) + p(\frac{x}{2})} & y = \frac{x}{2} \\[2ex] 0 & \text{else} \end{cases}$$

so $E(Y \mid X = x) = (2x)\dfrac{p(x)}{p(x)+p(\frac{x}{2})} + \left(\dfrac{x}{2}\right)\dfrac{p(\frac{x}{2})}{p(x)+p(\frac{x}{2})}$

eqn(2) ✓

$(a \mid B) = \{ a_1, a_2, \cancel{a_3} \}$

action space

offer to trade, offer accepted

offer to trade, offer refused

don't offer to trade

W = amount of new money you have after game concludes

$U(x) = ax + b \quad (a > 0)$

under a_1, $W = Y$

under a_2, $W = X$

~~conditional~~ expected utility under a_1, given $X = x$

conditional expected utility under a_2, given $X = x$

$E[\cancel{U(W)}] = E[\cancel{U(Y)}]$

$E[U(W) \mid X = x] = \cancel{E[U(X) \mid X = x]}$

$E[U(W) \mid X = x] = E(U(Y) \mid X = x)$

$(a > 0)$

$u(x) = \underline{\underline{a}}x + b$

$E[u(X) \mid X = x] =$

$= E[\underline{\underline{aX + b}} \mid X = x]$ a_2

$= E(ax + b) = ax + b$

a_1

$E[u(W) \mid X = x] =$

$E[u(X) \mid X = x] = E(aX + b \mid X = x)$

$= a\,E(X \mid X = x) + b$

a_1 is better than a_2 under

MEU iff $E[u(W) \mid X = x]$ under a_1 $>$ $E[u(W) \mid X = x]$ under a_2

iff $a\,E(X \mid X = x) + b > ax + b$

iff $a\,E(X \mid X = x) > ax$

iff $E(X \mid X = x) > x$ iff

$$(2x)\frac{p(x)}{p(x) + p(\frac{x}{2})} + (\frac{x}{2})\frac{p(\frac{x}{2})}{p(x) + p(\frac{x}{2})} > x$$

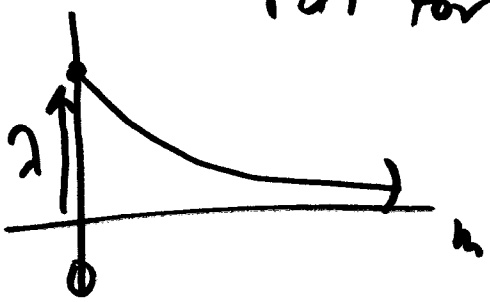
iff $$\frac{2\,p(x)}{p(x)+p(\frac{x}{2})} + \frac{\frac{1}{2}p(\frac{x}{2})}{\cancel{2}\left(p(x)+p(\frac{x}{2})\right)} > 1$$

iff $$2\,p(x) + \frac{1}{2}p(\frac{x}{2}) > p(x) + p(\frac{x}{2})$$

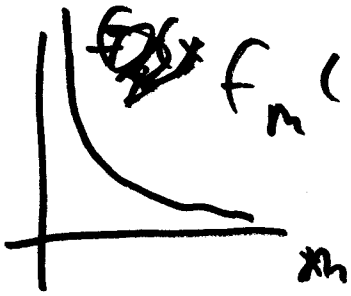
iff $$p(x) > \frac{1}{2}p(\frac{x}{2})$$

iff $$2\,p(x) > p(\frac{x}{2}) \quad \checkmark$$

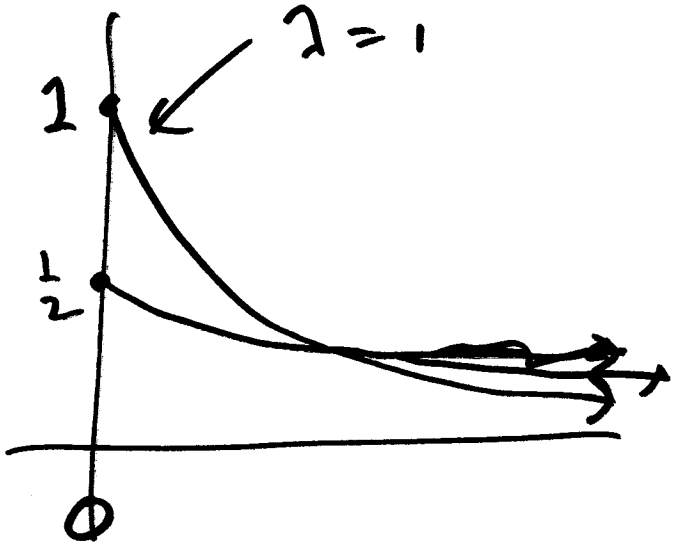
PDF for M | example of a decreasing PDF — part of B



$$p(m) = f_M(m) \quad (\lambda > 0)$$
$$= \begin{cases} \lambda e^{-\lambda m} & \text{for } m > 0 \\ 0 & \text{else} \end{cases}$$



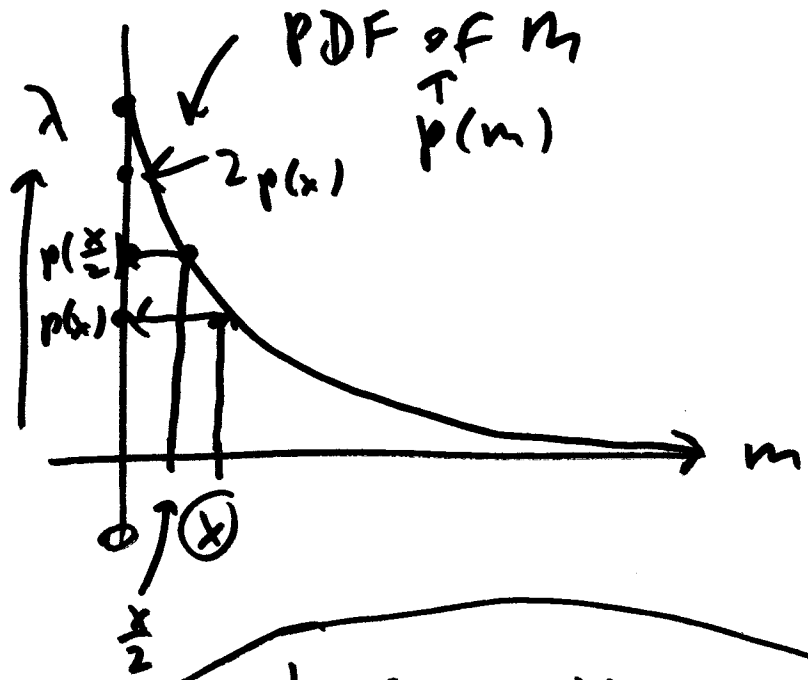
$$f_M(m) = \begin{cases} \frac{1}{m^2} & \text{for } m > 1 \\ 0 & \text{else} \end{cases}$$



$$p(m) = f_M(m) = \begin{cases} \lambda e^{-\lambda m} & \text{for } m \geq 0 \\ 0 & \text{else} \end{cases}$$

λ controls center, spread, skewness, tail weight

$$\begin{pmatrix} \text{smaller} \\ \lambda \end{pmatrix} \leftrightarrow \begin{pmatrix} \text{more probability} \\ \text{on big values of } M \end{pmatrix}$$



$$\begin{pmatrix} q_1 \text{ is better} \\ \text{than } q_2 \end{pmatrix}$$

$$\updownarrow$$

$$\begin{pmatrix} \text{you should} \\ \text{offer to trade} \end{pmatrix}$$

iff

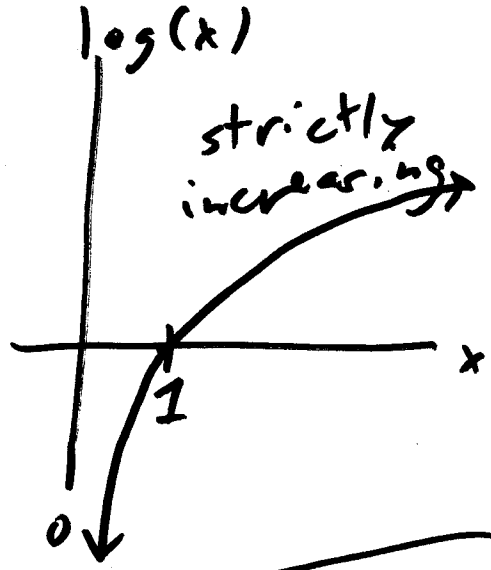
$$\boxed{p\left(\frac{x}{2}\right) < 2\,p(x)}$$

basic qualitative answer

offer to trade iff x is small

for $m \geq 0$

$$p(m) = \lambda e^{-\lambda m}$$



if $(M|\lambda) \sim$ Exponential(λ)

then $E(M|\lambda) = \frac{1}{\lambda}$

offer to trade iff

$$p\left(\frac{x}{2}\right) < 2\,\underline{\underline{p(x)}}$$

iff

$$\lambda e^{-\lambda \frac{x}{2}} < 2\lambda e^{-\lambda x}$$

iff

$$e^{-\frac{\lambda x}{2}} < 2 e^{-\lambda x}$$

iff

$$-\frac{\lambda x}{2} < \log 2 - \lambda x$$

iff

$$-\lambda x < 2\log 2 - 2\lambda x$$

iff

$$2\lambda x - \lambda x < 2\log 2$$

$$\lambda x < 2\log 2$$

offer to trade if x is small $\longleftrightarrow$ $x < \frac{2\log 2}{\lambda} = 1.39\,E(M|\lambda) = 2\log 2\,E(M|\lambda)$