Before the game begins, you have uncertainty about the following:

- \( m \): amount referee puts into \( E_1 \) (\( m \))
- \( x \): amount you see in your \( E \) (\( x \))
- \( y \): amount your opponent sees in her/his/their \( E \) (\( y \))

\[ E = \begin{cases} 1 & \text{if you got } E_1 \\ 2 & \text{if not} \end{cases} \]

\[ P(\mathbb{E} = y \mid \mathbb{E} = x) = ? \]

So partition over another unknown such that, if it were known, \( \Theta \) would be easier partitioning over \( \mathbb{E} \) is messy & difficult (\( \Theta \))

So let's partition over \( \mathbb{E} = 1 \) or 2

\[ P(\mathbb{E} = y \mid \mathbb{E} = x) = P(\mathbb{E} = y, \mathbb{E} = 1 \mid \mathbb{E} = x) + P(\mathbb{E} = y, \mathbb{E} = 2 \mid \mathbb{E} = x) \]

\[ = P(\mathbb{E} = y \mid \mathbb{E} = 1, \mathbb{E} = x) \cdot P(\mathbb{E} = 1 \mid \mathbb{E} = x) + P(\mathbb{E} = y \mid \mathbb{E} = 2, \mathbb{E} = x) \cdot P(\mathbb{E} = 2 \mid \mathbb{E} = x) \]
and now, several excellent things happen:

1. Given \( E = x \), \( (E = 1) = (M = x) \) (why?)
and \( (E = 2) = (M = \frac{x}{2}) \) (why?)

2. Given \( (E = 1, Z = x) \)
the only possible value for \( Z \) is \( y = 2x \) (why?)
and given \( (E = 2, Z = x) \) the only possible value for \( Z \) is \( y = \frac{x}{2} \) (why?)

so the conditional distribution \( P(Z = y | E = x) \) looks like this:

Fix on \( x \in S_E^2 \): The support of \( (Z | E = x) \) is just the 2 points \( \{ 2x, \frac{x}{2} \} \) (from 2 above)

so \( P(Z = 2x | E = x) = \frac{1}{2} P(E = 2, Z = x) \) + \( P(E = 1, Z = x) \cdot P(M = x | Z = x) \)

\[ = P(M = x | Z = x) \]
\[
\begin{align*}
\mathbb{P}(Z = \frac{x}{2} | X = x) &= \sqrt{\frac{1}{2} \mathbb{P}(Z = \frac{x}{2} | X = x)} \cdot \mathbb{P}(M = \frac{x}{2} | X = x) \\
&= \mathbb{P}(M = \frac{x}{2} | X = x) \\
&= \frac{\mathbb{P}(M = \frac{x}{2})}{\mathbb{P}(X) + \mathbb{P}(\frac{x}{2})} \\
\mathbb{P}(M = 2x | X = x) &= \mathbb{P}(M = x | X = x) \\
&= \frac{\mathbb{P}(M = x)}{\mathbb{P}(X) + \mathbb{P}(\frac{x}{2})} \\
\int_{x \in \mathbb{R}} f_{\mathbb{P}(X \mid Z)}(y | x) = \\
\begin{cases} \\
\frac{\mathbb{P}(x)}{\mathbb{P}(x) + \mathbb{P}(\frac{x}{2})} & \text{if } y = 2x \\
\frac{\mathbb{P}(\frac{x}{2})}{\mathbb{P}(x) + \mathbb{P}(\frac{x}{2})} & \text{if } y = \frac{x}{2} \\
0 & \text{else}
\end{cases}
\end{align*}
\]
\[ E(\Xi | \Xi = x) = (2x) \frac{p(x)}{p(x) + p(\xi)} + (\frac{x}{2}) \frac{p(\xi)}{p(x) + p(\xi)} \]

\[ e_{2x}(z) \checkmark \]

\[ (A | B) = \{ a_1, a_2 \} \]

- Action space
- Offer trade, offer refused or accepted
- Don't offer to trade
- New amount of money you have after game concludes
- Conditional expected utility under \( a_1 \), \( W = x \)
- Conditional expected utility under \( a_2 \), \( \bar{W} = x \)
- \( E[U(\xi)] = E[U(\Xi)] \)
- \( E[U(W) | \Xi = x] = E[U(W) | \Xi = x] \)
\[
(q > 0) \quad \text{if} \quad U(x) = qx + b \quad \Rightarrow \quad E[U(x) | X = x] = E[qX + b | X = x] = E[qX + b] = qx + b
\]

\[E[U(W) | X = x] = E[qX + b | X = x] = qx + b \quad \text{for all } q, \text{ is better under } q_1 \text{ than } q_2 \text{ under } q_2 \text{ un} \quad \text{iff} \quad E[U(W) | X = x] > \text{ under } q_1, \quad E[U(W) | X = x] \quad \text{under } q_2
\]

\[q \quad \text{iff} \quad qE(X | X = x) + b > qx + b
\]

\[q \quad \text{iff} \quad E(X | X = x) > qx
\]

\[q \quad \text{iff} \quad E(X | X = x) > x
\]

\[(2x) \frac{y(x)}{p(x) + y(x)} + (\frac{x}{2}) \frac{y(x)}{p(x) + y(x)} > x
\]
\[
\frac{2 \ p(x)}{p(x) + p(\frac{x}{2})} + \frac{\frac{1}{2} p(\frac{x}{2})}{\frac{1}{2} [p(x) + p(\frac{x}{2})]} > 1
\]

\[
2 \ p(x) + \frac{1}{2} \ p(\frac{x}{2}) > p(x) + p(\frac{x}{2})
\]

\[
p(x) > \frac{1}{2} p(\frac{x}{2})
\]

\[
2 \ p(x) > p(\frac{x}{2}) \quad \checkmark
\]
The variable $m$ has a probability density function (PDF) given by $p(m) = \lambda e^{-\lambda m}$ for $m \geq 0$ and 0 otherwise. This PDF controls the spread, skewness, and tail weight.

A smaller $\lambda$ corresponds to a probability distribution that gives more weight to big values of $m$.

The inequality $p(\frac{x}{2}) < 2p(x)$ would tell you that if $x$ is small, then $\frac{x}{2}$ is also small.
for \( n \geq 0 \)

\[ p(n) = 2e - \frac{2^n}{n!} \]

\[
\log(x)
\]

\[
x \quad \text{strictly increasing}
\]

if \((m12) \sim \exp(\lambda)\)

then \(E(m12) = \frac{1}{\lambda}\)

offer to trade if

\[ x \text{ is small} \]

offer to trade if

\[ \frac{x}{2} < 2p(x) \]

\[ \frac{x}{2} < 2e - \frac{2^x}{x!} \]

\[ e^{-\frac{x}{2}} < 2e^{-2x} \]

\[ 2e^{-\frac{x}{2}} < 2 \log 2 - 2x \]

\[ 2x - 2x < 2 \log 2 \]

\[ 2x < 2 \log 2 \]

\[ x < \frac{2 \log 2}{2} = \log 2 \]