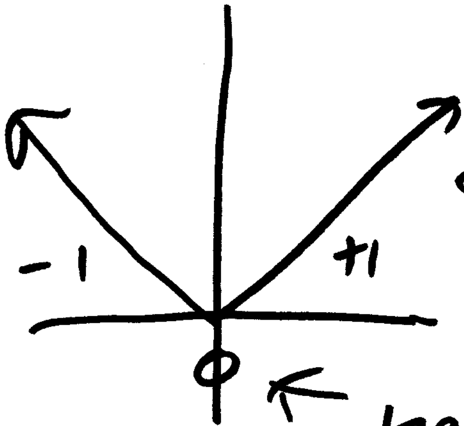
STAT 131

21 May 20

(DJ office 1.5 hour)




continuous everywhere

not differentiable at 0

STAT 2 #3(a) (1)

$X \sim$ Binomial$(n, p)$

$0 < p < 1$

$n \geq 3$

$E(X^1) = np = \mu_X$

$E(X^2) = np[1 + (n-1)p]$

$E(X^3) =$

$$np[1 + (n-1)(n-2)p^2 + 3p(n-1)]$$

$V(X) = np(1-p) = \sigma_X^2$

$X$ has PMF

$$f_X(x \mid n, p) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x} & \text{for } x = 0, 1, \ldots, n \\ 0 & \text{else} \end{cases}$$

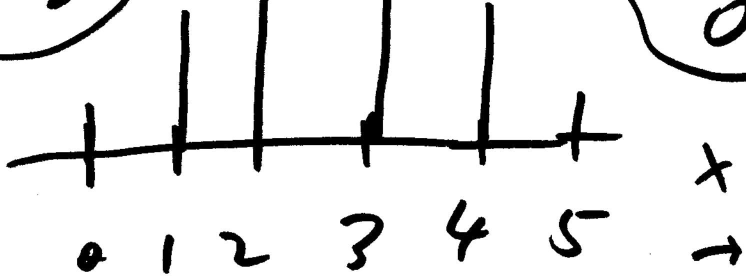
$n = 5$
$p = \frac{1}{4}$

$p < \frac{1}{2}$

T-S

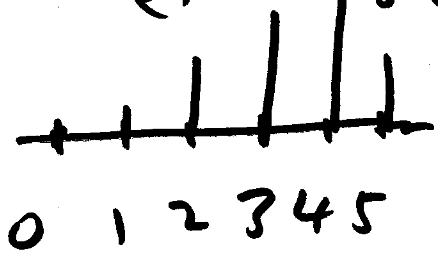

skewness > 0

$n=5$, $p=\frac{1}{2}$ → skewness $0$




symmetric about $x = 2.5$

$(n=5, p=\frac{3}{4})$
(skewness $< 0$)




mirror image of $(n=5, p=\frac{1}{4})$ reflected about $x = 2.5$

$$\text{skewness}(X) = E\left[\left(\frac{X-\mu_X}{\sigma_X}\right)^3\right]$$

$$= \frac{1}{\sigma_X^3} E\left[(X-\mu_X)^3\right]$$

$$= \frac{1}{\sigma_X^3} E\left(X^3 - 3X^2\mu_X + 3X\mu_X^2 - \mu_X^3\right)$$

$$SD(X) = \sigma_X$$
$$= \sqrt{V(X)}$$
$$= \sqrt{np(1-p)}$$

$$\text{skewness}(X) = \frac{1}{\sigma_X^3}\left\{ E(X^3) + E(\underline{\underline{-3\mu_X}}\, X^2) + E(\underline{3\mu_X^2}\, X) + E(\underline{-\mu_X^3}) \right\}$$

$$= \frac{1}{\sigma_X^3}\left\{ E(X^3) - 3\mu_X E(X^2) + 3\mu_X^2 E(X) - \mu_X^3 \right\}$$

(hideous algebra)

$$= \frac{1-2p}{\sqrt{np(1-p)}} = 0 \longleftrightarrow 1 - 2p = 0 \longleftrightarrow p = \frac{1}{2}$$

plugging in exp. for $E(X)$, ...

proof by $\boxed{\text{wd}}$

all Binomial distributions with $p = \frac{1}{2}$ are symmetric, no matter what $n$ is

④

$$\text{skewness}(X) = \frac{1-2p}{\sqrt{np(1-p)}} \rightarrow 0$$

as $n \rightarrow \infty$ for all $0 < p < 1$

$$\text{skewness}(X) \rightarrow 0 \text{ as } n \rightarrow \infty$$

discrete

$(X \mid \lambda) \sim \text{Poisson}(\lambda) \longleftrightarrow$

$(\lambda > 0)$

$$f_X(x \mid \lambda) = \begin{cases} \dfrac{\lambda^x e^{-\lambda}}{x!} & \text{for } x = 0, 1, \ldots \\ 0 & \text{else} \end{cases}$$

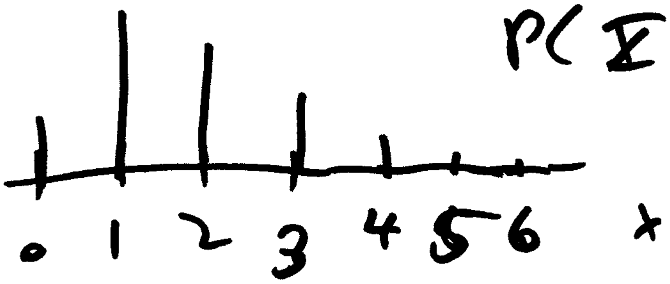
$\quad \| $

$P(X = x \mid \lambda)$

PMF

$$S_X = \{0, 1, \ldots\}$$

Poisson can model $X = \begin{pmatrix} \text{\# of interesting} \\ \text{occurrences} \\ \text{in } (0, T) \end{pmatrix}$




$P(X = x \mid \lambda = 1.1)$

PMF

$X$ discrete

MGF?

$$\psi_X(x) = E(e^{tX})$$

(LOTUS)

$$= \sum_{x=0}^{\infty} e^{tx} \cdot P(X = x \mid \lambda)$$

$$= \sum_{x=0}^{\infty} e^{tx} \cdot \frac{\lambda^x e^{-\lambda}}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{e^{tx} \cdot \lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda e^t)^x}{x!} = e^{-\lambda} \exp(\lambda e^t)$$

for any $w \in \mathbb{R}$

$$e^w = \sum_{x=0}^{\infty} \frac{w^x}{x!}$$

$$= e^{-\lambda} e^{\lambda e^t} = e^{-\lambda + \lambda e^t} = e^{\lambda(e^t - 1)} \checkmark$$