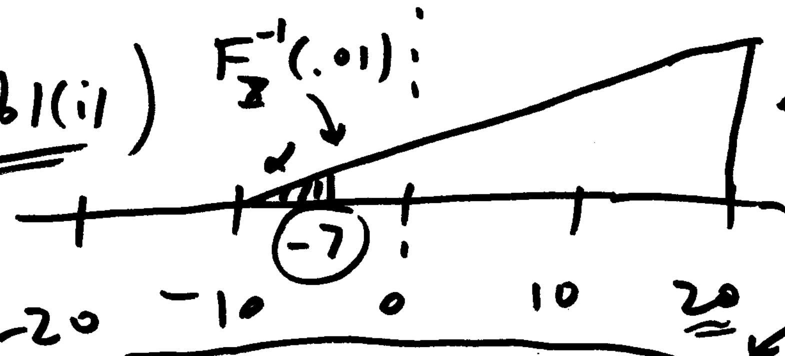
(5(b)(ii))



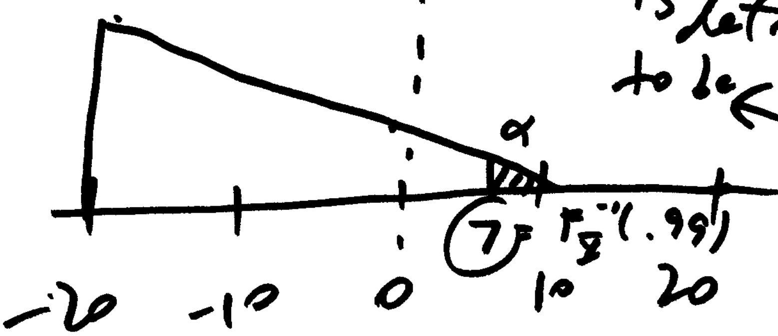
← PDF of X $f_X(x)$

STAT 131 2 May 20

DD extra office - 1.5 hour session

$\text{VaR} = V = -F_X^{-1}(\alpha) \doteq F_Y^{-1}(1-\alpha)$

is defined to be $Y \triangleq -X$ (1)



PDF $f_Y(y)$ of Y

'VaR = v = \$7M

$\text{VaR} = (1-\alpha)$ quantile of $Y = F_Y^{-1}(\alpha)$

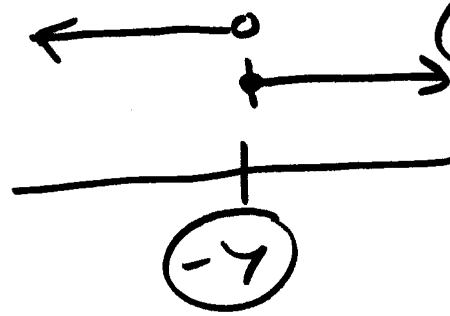
for some small positive α (e.g., .01)

$(\text{VaR} = v) \leftrightarrow F_Y(v) = 1-\alpha$

$F_Y^{-1}[F_Y(v)] = F_Y^{-1}(1-\alpha) = v$

show that $v = -F_X^{-1}(\alpha)$

$$F_Y(y) = P(Y \le y)$$



CDF

$$= P(-X \le y)$$

$$= P(X \ge -y) = \circledast$$

$$F_X(x) = P(X \le x)$$

Now $P(X < -y) + P(X \ge -y) = 1$

~~so $P(X \ge -y) = 1 - P(X < -y)$~~

$$F_Y(y) = P(X \ge -y)$$

$$= 1 - P(X < -y)$$

X is continuous so $P(X = -y) = 0$

$$F_Y(y) = 1 - P(X \le -y)$$

$$= 1 - F_X(-y)$$

③

$(VaR = v) \overset{def}{\iff} F_Y(v) = 1-\alpha$

$= 1 - F_X(-v)$

$1-\alpha = 1 - F_X(-v)$

$-\alpha = -F_X(-v)$

$\alpha = F_X(-v)$

$F_X^{-1}(\alpha) = F_X^{-1}\left[F_X(-v)\right]$

$= -v$ so

$v = VaR = -F_X^{-1}(\alpha)$ ✓

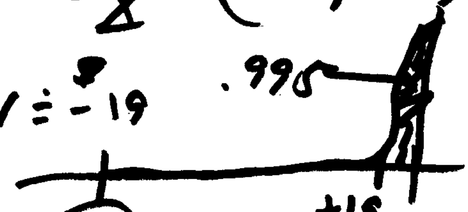
X continuous r.v.

$F_X(x) = P(X \le x) = \int_{-\infty}^{x} f_X(t)\,dt$

PDF	VaR ($M)
5(b)(i)	$7M
5(b)(ii)	✓
5(b)(iii)	✓
5(c)(i)	✓
5c(ii)	$9.85M

not variance

$S_X = (-10, +20)$



PDF of X

-10 $\quad$ $+20$

yesterday I claimed

~~$0 \le$~~ $\boxed{V \le 10}$

$-20 \le$

what they didn't tell you in elementary school

All math can do to help us solve real-world problems is to give us statements of the form

If A then C

A ← assumptions

C ← conclusions

that we can count on

people don't want If A then C ;

they want C

the best possible outcome from using math to decide what to do in the world is: suppose we can show that

$$\left\{ \begin{array}{l} \text{If } A_1 \text{ then } \underline{\underline{C}} \\ \text{If } A_2 \text{ then } \underline{\underline{C}} \\ \vdots \\ \text{If } \underline{A_n} \text{ then } \underline{\underline{C}} \end{array} \right.$$

sensitivity analysis

(<u>last</u> <u>assumption</u>) $A_1, \ldots, A_n$ are the only reasonable assumptions to ~~[illegible]~~ consider $\longrightarrow$ then C to show: $P(A \mid B_i) = \begin{cases} 0 & \text{for } i \le r \\ \dfrac{r}{i-1} & i > r \end{cases}$

we hire best

B_i: the best person is in interviewing slot i

for fixed n (positive integer) and fixed $r = 0, 1, \dots, n$

non-negative integer

$n = 10$

$r = 4$

$i = \cancel{8}\ 7$

$i = 7$ → $(n = 10)$

31 87 79 50 | — — — — — —

quality pool   hiring pool

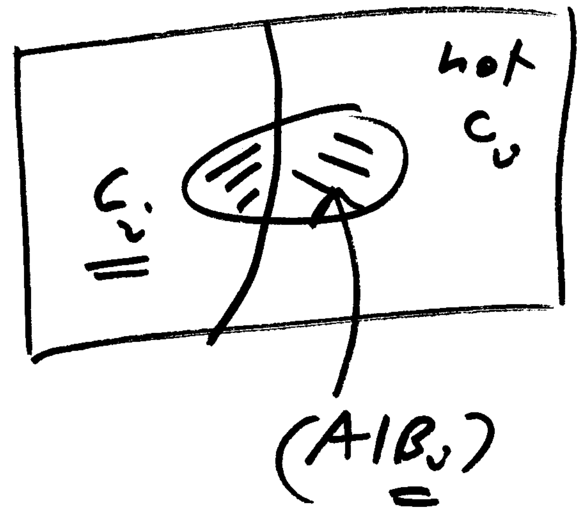
← r →

C_i = (we're still interviewing as of slot i)

quality scores: $[0, 100]$

we interview candidates in random quality order for $i > r$ $\underline{\underline{P(A \mid B_i)}} = ?$ (hard)

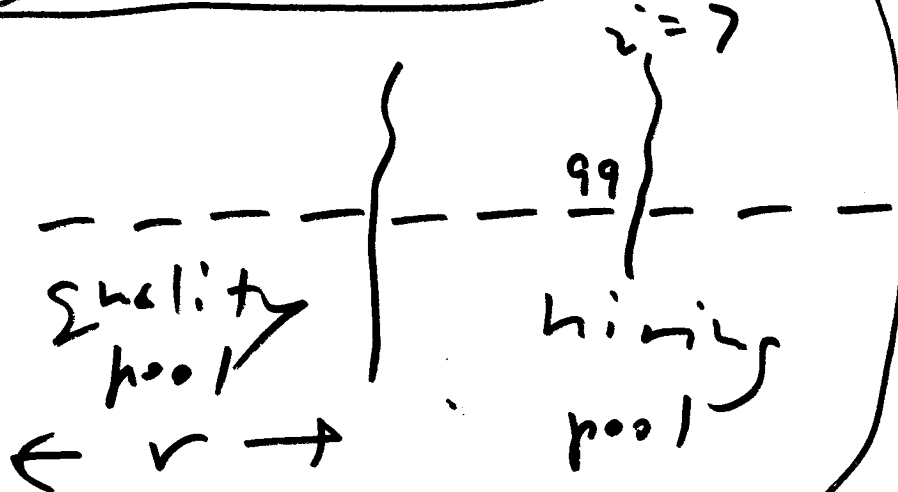
so: let's try bringing C_i into the calculation



$\underline{\underline{P(A \mid B_i)}} =$

$P(A \text{ and } C_i \mid B_i) +$

$\cancel{P(A \text{ and not } C_i \mid B_i)}$ 0



$P(A \mid B_i) =$

$P(A \text{ and } C_i \mid B_i)$

$P(E \text{ and } F) = P(E) \cdot P(F \mid E)$

$= P(F) \cdot P(E \mid F)$

P(A and C_i | B_i

$$P(E \text{ and } F \mid G) = \cancel{P(\overset{A}{E} \mid \overset{B_i}{G}) \cdot P(\overset{C_i}{F} \mid \overset{A,}{E}, \overset{B_i}{G})}$$

and

$$= P(\overset{C_i}{F} \mid \overset{B_i}{G}) \cdot P(\overset{A}{E} \mid \overset{C_i}{F}, \overset{B_i}{G})$$

$$= P(A \mid B_i) = P(A \text{ and } C_i \mid B_i)$$

$$= P(C_i \mid B_i) \boxed{\cancelto{1}{P(A \mid C_i, B_i)}}$$

$$= P(C_i \mid B_i) = P(\text{2nd best person has to be in QP} \mid B_i) = \frac{r}{i-1}$$ ✓

n = 10

r = 4

i = 7

(no ties)

31 87 79 50 | ~~94~~ _ 98 _ _ _ _

Quality pool (QP) | hiring pool