STAT 131
2 Jun 20

DD office
1.5-hour
session

Quiz 10

to

$$
P = \begin{array}{c|c}
 & \begin{array}{ccc} xy & xz & yz \end{array} \\
\begin{array}{c} xy \\ xz \\ yz \end{array} & \begin{bmatrix} 0 & p_{xy} & 1-p_{xy} \\ p_{xz} & 0 & 1-p_{xz} \\ p_{yz} & 1-p_{yz} & 0 \end{bmatrix}
\end{array}
$$

from

$p_{xy} = P(X \text{ beats } Y)$

$p_{xz} = P(X \text{ beats } Z)$

$p_{yz} = P(Y \text{ beats } Z)$

$$xy \to xz \to xy \to \underbrace{yz \to xz}_{\downarrow \; xy}$$

Quiz 8

$u_2(x) = 1 + \log(x)$

THT 3

1(a)

$V(\bar{e}_n) = \,?$

$e_i \overset{\text{IID}}{\sim}$ $E(e_i) = 0$

$\underline{V(e_i)} = \sigma^2$

(2)

$$V(\bar{e}_n) = V\left(\frac{1}{n}\sum_{i=1}^{n} e_i\right)$$

$$= \left(\frac{1}{n}\right)^2 V\left(\sum_{i=1}^{n} e_i\right) = \frac{\sigma^2}{n} \checkmark$$

$$\overset{\text{IID}}{=} \frac{1}{n^2}\sum_{i=1}^{n} \underline{V(e_i)} = \frac{1}{n^2}\sum_{i=1}^{n}\sigma^2 = \frac{n\sigma^2}{n^2}$$

PDF of e_i

$E(e_i) = 0$

$V(e_i) = E\left[e_i - E(e_i)\right]^2$

$$Y_i = \theta + b + e_i$$

θ, b: constants

e_i: r.v., IID

Y_i: r.v.

$E(Y_i) =$

$E(\theta + b + e_i)$

$= \theta + b + E(e_i)$

θ: truth; b: bias

suppose temporarily that $E(e_i) = \underline{1}$

$$E(Y_i) = \theta + \underbrace{(b + 1)}_{b^* = b + 1} = \theta + b^*$$

θ: truth; b^*: bias

Quiz 8 (cont) | $U_2(x) = 1 + \log(x)$ | Support of $X = \{A-B, A+B\}$ ③

$$f_X(x) = \begin{cases} p & \text{if } x = \underline{A+B} \\ 1-p & x = \underline{A-B} \\ 0 & \text{else} \end{cases}$$

PMF

$$E[U_2(X)] = E[\underline{1} + \underline{\log(X)}]$$

$$= 1 + E[\log(X)]$$

$$= 1 + p \cdot \underline{\log(A+B)} + (1-p) \cdot \log(A-B)$$

$A = 10$

$B = (-5, A)$

$p = \{0.4, 0.7\}$

$$\frac{d}{dB} E[U_2(X)] =$$

$$\frac{d}{dB}\left[1 + p\log(A+B) + (1-p)\log(A-B)\right]$$

4

$$\frac{\partial}{\partial B}\left[E\, u_2(X)\right] = \frac{A(2p-1)-B}{(A-B)\cancel{(A+B)}} = 0$$

$(0 \le B < A)$

& solve for $B \rightarrow$

$$B^* = A(2p-1) = 2A(p-\tfrac{1}{2}) = 2(p-\tfrac{1}{2})A$$

$(A=10,\ p=0.7) \rightarrow B^* = 20(0.7-0.5)$

$= 4$ ✓