STAT 131
19 May 20

~~Office~~ hour

THT 2

#2(a)

$\underline{69} \cdot \underline{68} \cdot \underline{67} \cdot \underline{66} \cdot \underline{65} \mid \cdot \underline{26}$

w   r

35 billion

(1)

$P(\text{grand prize}) = P(\text{get all 5 w correct } \underline{\underline{\text{and}}} \text{ get R correct})$

$= P(\text{get all 5 w right}) \cdot P(\text{get R right})$

5 correct, 1

64 incorrect, 1, 69

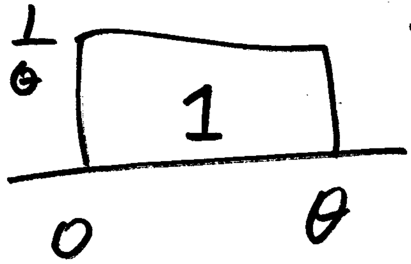
$$= \frac{\cancelto{1}{\binom{5}{5}}\cancelto{1}{\binom{64}{0}}}{\binom{69}{5}} \cdot \frac{\cancelto{1}{\binom{1}{1}}\cancelto{1}{\binom{25}{0}}}{\binom{26}{1}}$$

$\binom{26}{1} \to 26$

$$= \frac{1}{\binom{69}{5} \cdot 26} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{69 \cdot 68 \cdot 67 \cdot 66 \cdot 65 \cdot 26}$$

35 billion

THT 2 #4



PDF of Y_i

$f_{Y_i}(y_i \mid \theta) =$

$$\frac{1}{\theta} I(0 < y_i < \theta) = \begin{cases} \frac{1}{\theta} & \text{for } 0 < y_i < \theta \\ 0 & \text{else} \end{cases}$$

$(0 < y_i < \theta$ for all $i = 1, \ldots, n)$

$\updownarrow$

$= (0 < \underbrace{\max(y_i)}_{m} < \theta)$

$\mathbf{y} = (y_1, \ldots, y_n)$

$\mathbf{Y} = (Y_1, \ldots, Y_n)$ ← vector (bold)

$f_{\mathbf{Y}}(\mathbf{y} \mid \theta) = P($ prob. beh. of Y_1 and Y_2 and $\ldots$ and $Y_n \mid \theta)$

$\overset{\text{indep.}}{=} P(Y_1 \mid \theta) \cdot P(Y_2 \mid \theta) \cdots P(Y_n \mid \theta)$

$(Y_i \mid \theta) \overset{IID}{\sim} \text{Uniform}(\theta)$
$(i = 1, \ldots, n)$

$$\circledast = \frac{1}{\theta} I(0 < y_1 < \theta) \, \frac{1}{\theta} I(0 < y_2 < \theta) \cdots \frac{1}{\theta} I(0 < y_n < \theta)$$

$$\circledast = \frac{1}{\theta^n} I(0 < y_1 < \theta) \cdot I(0 < y_2 < \theta) \cdot \ldots \cdot I(0 < y_n < \theta)$$

$$= \frac{1}{\theta^n} I(0 \le y_1 \le \theta \text{ and } 0 \le y_2 \le \theta \text{ and} \ldots \text{ and } 0 \le y_n \le \theta)$$

$$= \frac{1}{\theta^n} I(\overset{\max(y_1, \ldots, y_n)}{m} \le \theta) = f_{\underset{\sim}{Y}}(\underset{\sim}{y} \mid \underline{\theta})$$

this looks like a function of $\underset{\sim}{y}$ for fixed θ, but it's really just a function of both $\underset{\sim}{y}$ and θ, and if we ~~want~~ were free to also think of it as a fn of θ for fixed $\underset{\sim}{y}$ Laplace (1774), Fisher (1922) under IID sampling

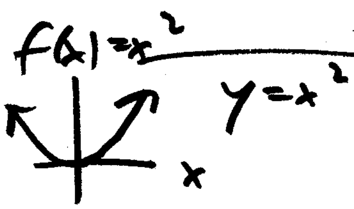
$$f_{\underset{\sim}{Y}}(\underset{\sim}{y} \mid \theta) = \prod_{i=1}^{n} f_{Y_i}(y_i \mid \theta)$$

$$\ell(\theta \mid \underset{\sim}{y}) = c \; f_{\underset{\sim}{Y}}(\underset{\sim}{y} \mid \theta)$$

likelihood function

$(c > 0)$ constant

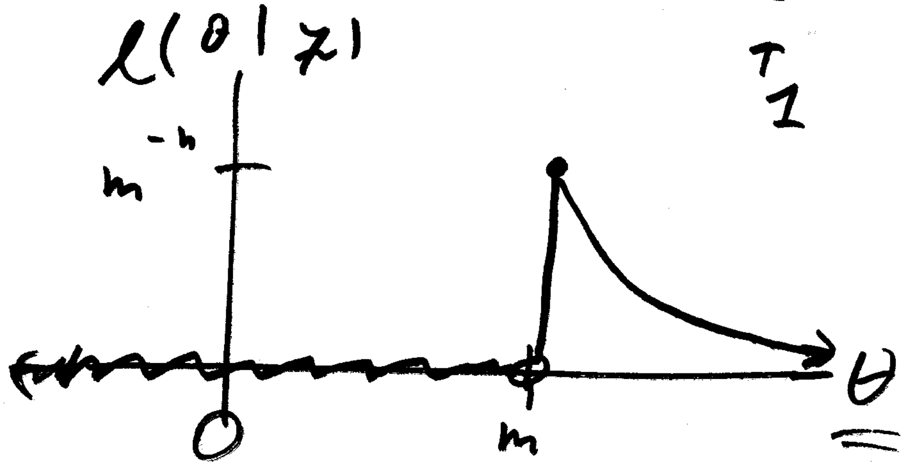
$$= c \, \frac{1}{\theta^n} \, I(m \le \theta)$$



$\theta \ge m$

$$= \underset{1}{c} \; \theta^{-n} \, I(m \le \theta)$$

$m = \max(y_1, \dots, y_n)$



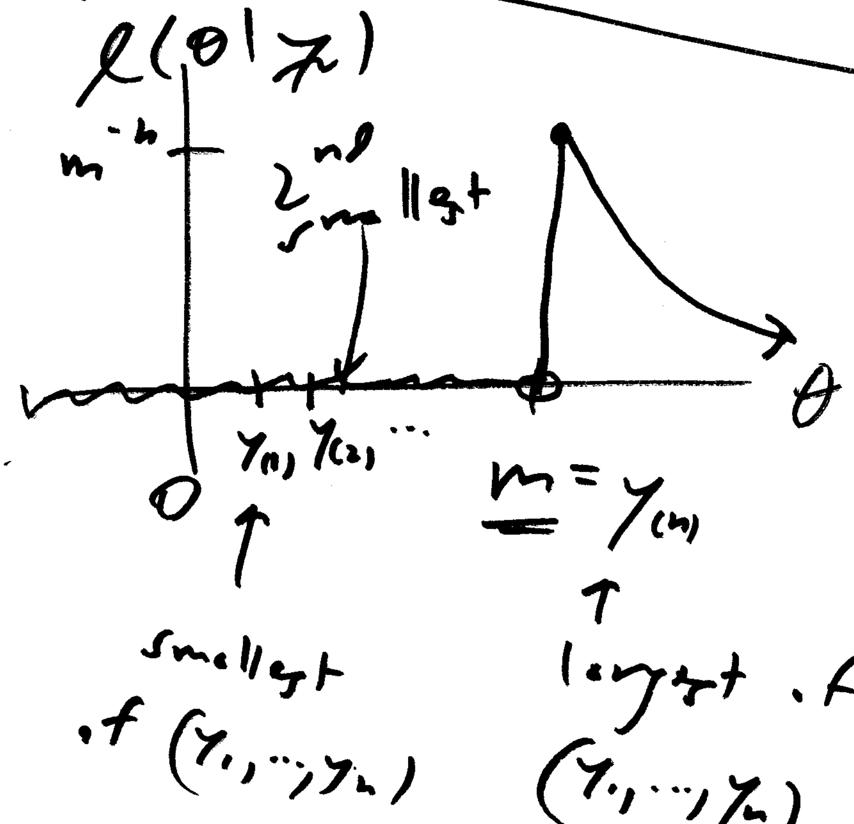
$$\hat{\theta}_{MLE} = m$$

Fisher: trust me, define

$\hat{\theta}_{MLE}$ = the θ that maximizes $\ell(\theta | y)$

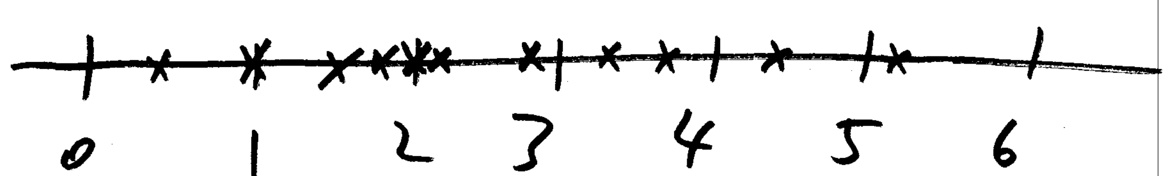
maximum likelihood estimate (or)

& use $\hat{\theta}_{MLE}$ as a good estimate of θ



$S_{Y_i} = (0, \theta)$

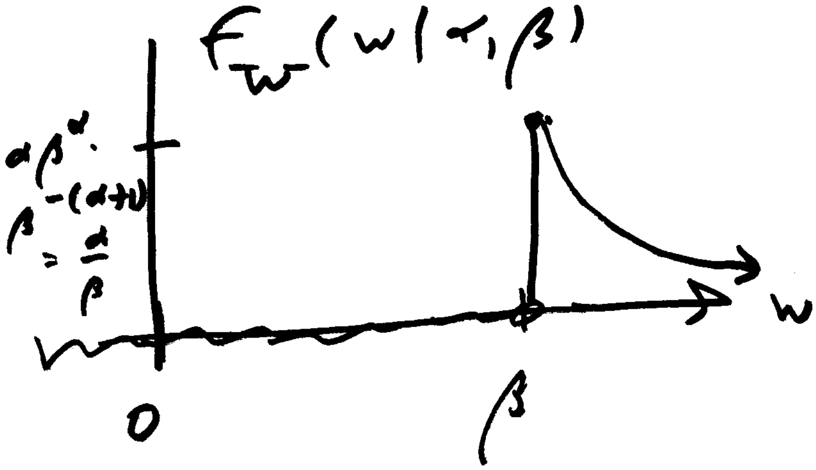
$(Y_i | \theta) \stackrel{IID}{\sim}$ Uniform$(0, \theta)$



$$\underset{\sim}{W} \sim \text{Pareto}(\alpha, \beta) \rightarrow \overset{\text{PDF}}{f_{\underset{\sim}{W}}(w \mid \alpha, \beta)} = \begin{cases} C\, w^{-(\alpha+1)} & \text{for } w \geq \beta \\ 0 & \text{else} \end{cases}$$

$(\alpha, \beta > 0)$

$C = (\alpha \beta^{\alpha})$



$$l(\theta \mid \underset{\sim}{x}) = \boxed{C}\, f_{\underset{\sim}{X}}(\underset{\sim}{x} \mid \theta)$$

$l(\theta \mid \underset{\sim}{x})$ is an un-normalized Pareto distribution in θ

$$\int_m^{\infty} \theta^{-n}\, d\theta = \frac{m^{1-n}}{n-1}$$

$$l(\theta \mid \underset{\sim}{x}) = \theta^{-n}\, I(\theta \geq m)$$

$$l_{\text{normalized}}(\theta \mid \underset{\sim}{x}) = \frac{n-1}{m^{1-n}}\, \theta^{-n}\, I(\theta \geq m)$$