STAT 131
30 Apr 20
DD office
1.5 hour
session

(1)

truth

what were decide	lots of new c-19	not
closed	✓	
open		✓

campus closed but didn't need to (not great)

big c-19 outbreak on ucsc campus (much worse)

THT 1 # 4(b)

show

$$P(A \mid B_i) = \begin{cases} 0 \quad \checkmark & i \le r \\ \frac{r}{i-1} & i > r \end{cases}$$

we hire best person

best person in slot i

$n = 8$

$r = 3$

$i = 5$

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qp — interview pool

$r = 3$

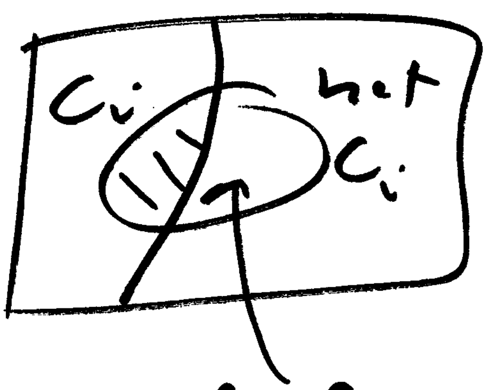
$i = 5$ quality pool

quality score in $[0, 100]$

C_i = (we are still interviewing as of candidate i)

$$\boxed{P(A \mid B_i) = ?}$$

partition over C_i vs. (not C_i)



$(A \mid B_i)$

$$p(A \mid B_i) =$$

$$P(A \text{ and } C_i \mid B_i) + \overset{0}{\cancel{\boxed{P(A \text{ and not } C_i \mid B_i)}}}$$

$$P(E \text{ and } F) = P(E)\,P(F \mid E) = P(F)\,P(E \mid F)$$

$$P(\underset{A}{E} \text{ and } \underset{C_i}{F} \mid \underset{B_i}{\underline{G}}) = P(\underset{A}{E} \mid \underset{B_i}{G})\,P(\underset{C_i}{F} \mid \underset{A,\ B_i}{E, G}) = P(\underset{C_i \mid B_i}{F \mid G})\,P(E \mid F, G)$$

$$P(A \text{ and } C_i \mid B_i) = P(C_i \mid B_i)\, \underbrace{P(A \mid C_i, B_i)}_{1}$$

$$P(A \mid B_i) = P(A \text{ and } C_i \mid \underline{B_i})$$

$$= P(C_i \mid \underline{B_i}) = P\left(\begin{matrix}\text{were still}\\ \text{interviewing}\\ \text{as of}\\ \text{slot } i\end{matrix} \;\middle|\; \begin{matrix}\text{best}\\ \text{person}\\ \text{in}\\ \text{slot } i\end{matrix}\right)$$

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$n = 8$ $\quad r = 3$ $\quad i = 5$

$$= P\left(\begin{matrix}\text{2nd}\\ \text{best}\\ \text{person}\\ \text{so far}\\ \text{is in}\\ \text{QP}\end{matrix} \;\middle|\; \begin{matrix}\text{best}\\ \text{person}\\ \text{in}\\ \text{slot } i\end{matrix}\right)$$

$$= \frac{r}{i-1} \quad \checkmark$$

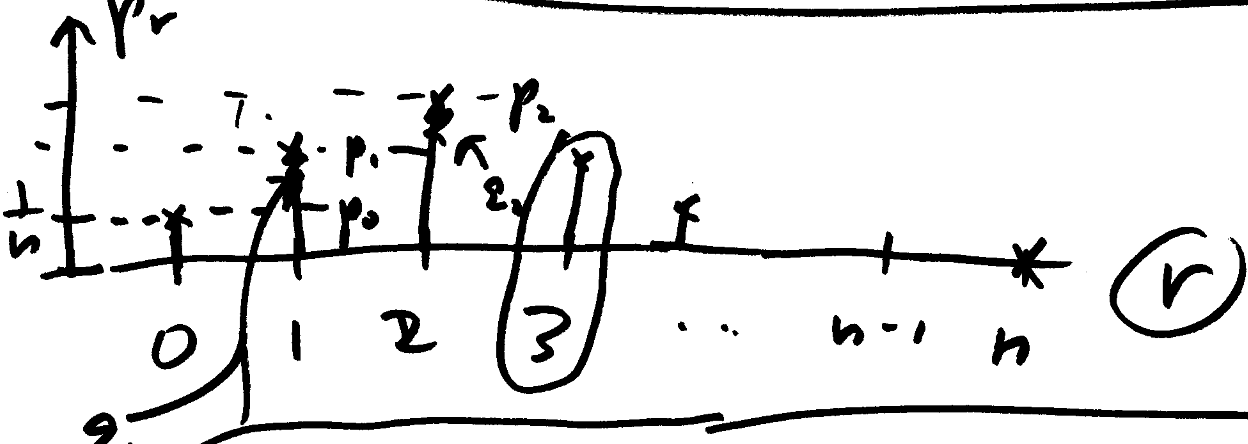
(c) $p_r = P(A$, having prespecified r before interviewing$)$

$$p_0 = \frac{1}{n}$$

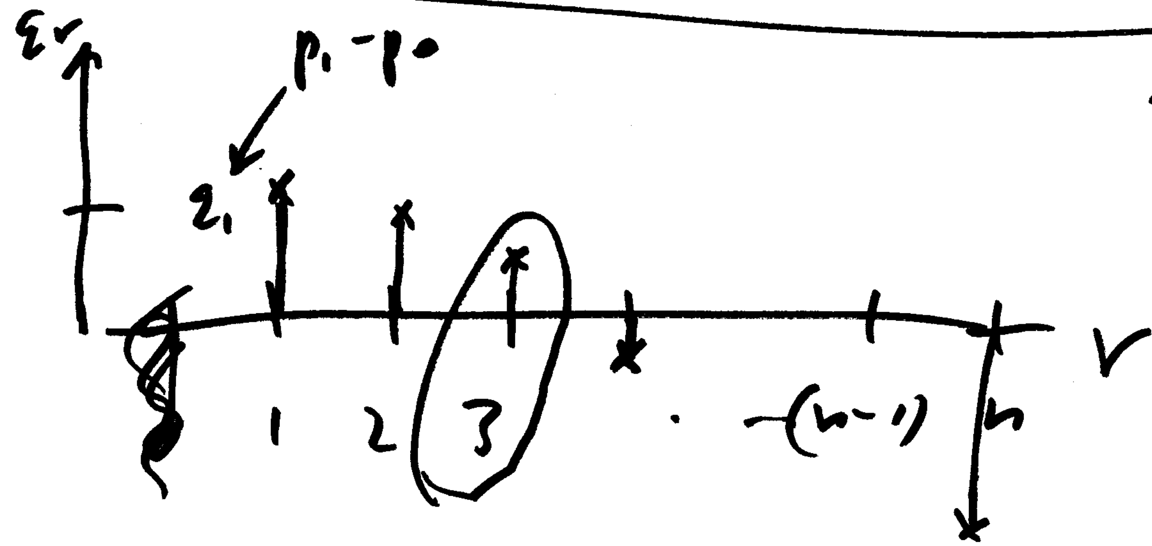
for $r = 1, \ldots, n-1$

$$p_r = \frac{r}{n} \sum_{i=r+1}^{n} \frac{1}{i-1}$$

(d) $\varepsilon_r = p_r - p_{r-1}$ for $r = 1, \dots, n$



$$\varepsilon_r = \frac{p_r - p_{r-1}}{r-(r-1)} \doteq \text{approximate derivative of function } p_r$$



to show: $\varepsilon_n < 0$

$p_0 = \frac{1}{n}$

$r = 1, \dots, n-1$ $\quad p_r = \frac{r}{n} \sum_{i=r+1}^{n} \frac{1}{i-1} = \frac{r}{n} S_{r,n}$

$$\varepsilon_1 = p_1 - p_0 = \left(\frac{1}{n} \sum_{i=2}^{n} \frac{1}{i-1} \right) - \frac{1}{n}$$

$$\varepsilon_2 = p_2 - p_1 = \left(\frac{2}{n}\sum_{i=3}^{n}\frac{1}{i-1}\right) - \left(\frac{1}{n}\sum_{i=2}^{n}\frac{1}{i-1}\right)$$

let's define $\displaystyle S_{r,n} = \sum_{i=r+1}^{n}\frac{1}{i-1}$

$$\varepsilon_1 = p_1 - p_0 = \frac{1}{n}S_{1,n} - \frac{1}{n}$$

$$\varepsilon_2 = p_2 - p_1 = \frac{2}{n}S_{2,n} - \frac{1}{n}S_{1,n}$$

$$p_r = \frac{r}{n}S_{r,n}$$

$$S_{1,n} = \sum_{i=2}^{n}\frac{1}{i-1}$$

$$= 1 + \frac{1}{2} + \dots + \frac{1}{n-1}$$

$$S_{2,n} = \sum_{i=3}^{n}\frac{1}{i-1}$$

$$= \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n-1}$$

$$S_{2,n} + 1 = S_{1,n}$$

as $r \uparrow$, $\delta_{r,n} \downarrow$ (for fixed n)

$$\varrho_2 - \varrho_1 = \left(\frac{2}{n} \delta_{2,n} - \frac{1}{n} \delta_{1,n} \right) - \left(\frac{1}{n} \delta_{1,n} - \frac{1}{n} \right)$$

$$= \frac{2}{n} \delta_{2,n} - \frac{2}{n} \delta_{1,n} + \frac{1}{n}$$

$\delta_{2,n} + 1 = \delta_{1,n}$

$$= \frac{2}{n} \left(\delta_{1,n} - 1 \right) - \frac{2}{n} \delta_{1,n} + \frac{1}{n}$$

$$(\varrho_2 - \varrho_1) = -\frac{2}{n} + \frac{1}{n} = -\frac{1}{n} < 0$$

$\varrho_2 < \varrho_1$ & similarly for $r = 2, \ldots, n-1$

$n = 10$

$$p_0 = \frac{1}{10}$$

$$p_r = \frac{r}{n} \sum_{i=r+1}^{n} \frac{1}{i-1}$$

$n = 10$

r	p_r
0	0.1
1	0.283
2	0.366
3	0.39869
4	0.398254
5	0.3728
6	0.327